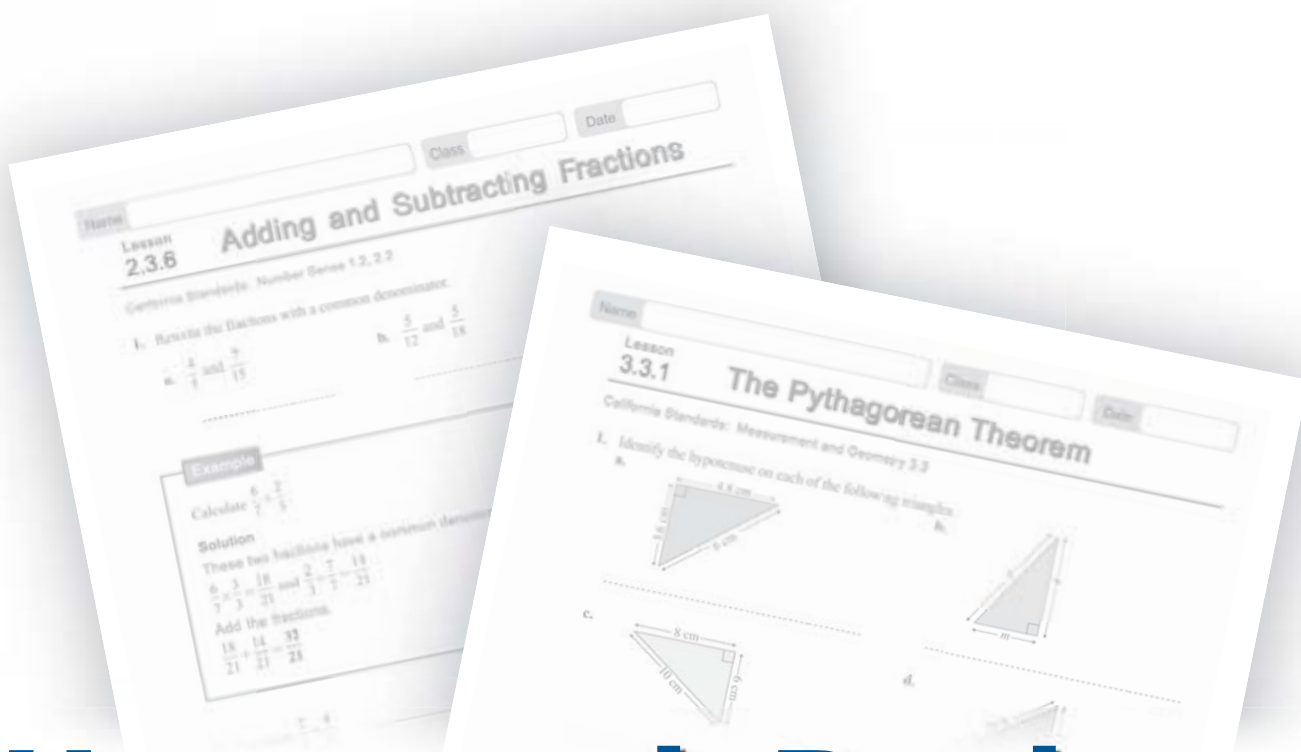


California Mathematics

Course Two



Homework Book

California Standards-Driven Program

California

Mathematics

Course Two

Homework Book

California Standards-Driven Program



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This book covers all of the requirements of the California Grade 7 Standards.

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California Grade Seven Mathematics Standards

The following table lists all the California Mathematics Content Standards for grade 7, with cross references to where each Standard is covered in this Program. This will enable you to measure your progress against the California Grade 7 Standards, as you work through the Program.

California Standard	Number Sense	Chapter
1.0	Students know the properties of, and compute with, rational numbers expressed in a variety of forms:	2, 5, 8
1.1	Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10), compare rational numbers in general.	2, 5
1.2	★ Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.	2, 5
1.3	Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.	2, 8
1.4	★ Differentiate between rational and irrational numbers.	2
1.5	★ Know that every rational number is either a terminating or a repeating decimal and be able to convert terminating decimals into reduced fractions.	2
1.6	Calculate the percentage of increases and decreases of a quantity.	8
1.7	★ Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.	8
2.0	Students use exponents, powers, and roots and use exponents in working with fractions:	2, 5
2.1	Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.	5
2.2	★ Add and subtract fractions by using factoring to find common denominators.	2
2.3	★ Multiply, divide, and simplify rational numbers by using exponent rules.	5
2.4	Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.	2
2.5	★ Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.	2
California Standard	Algebra and Functions	
1.0	Students express quantitative relationships by using algebraic terminology, expressions, equations, inequalities, and graphs:	1, 2, 3, 4, 5
1.1	Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).	1, 2, 3, 4
1.2	Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$.	1
1.3	★ Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.	1
1.4	Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.	1, 5
1.5	Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.	1, 4
2.0	Students interpret and evaluate expressions involving integer powers and simple roots:	2, 5
2.1	Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.	2, 5
2.2	Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.	5

California Grade Seven Mathematics Standards

3.0	Students graph and interpret linear and some nonlinear functions:	4, 5, 7
3.1	Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.	5
3.2	Plot the values from the volumes of three-dimensional shapes for various values of the edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and an equilateral triangle base of varying lengths).	7
3.3	★ Graph linear functions, noting that the vertical change (change in y -value) per unit of horizontal change (change in x -value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.	4
3.4	★ Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the ratio of the quantities.	4
4.0	★ Students solve simple linear equations and inequalities over the rational numbers:	1, 4
4.1	★ Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.	1, 4
4.2	★ Solve multistep problems involving rate, average speed, distance, and time or a direct variation.	4

California Standard

Measurement and Geometry

1.0	Students choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems:	3, 4, 7
1.1	Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).	4
1.2	Construct and read drawings and models made to scale.	3, 7
1.3	★ Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.	4
2.0	Students compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. They know how perimeter, area, and volume are affected by changes of scale:	3, 7
2.1	Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.	3, 7
2.2	Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.	3, 7
2.3	Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.	7
2.4	Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or $[1 \text{ ft.}^2] = [144 \text{ in.}^2]$, 1 cubic inch is approximately 16.38 cubic centimeters or $[1 \text{ in.}^3] = [16.38 \text{ cm}^3]$).	7
3.0	Students know the Pythagorean theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures:	3, 7
3.1	Identify and construct basic elements of geometric figures (e.g., altitudes, mid-points, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.	3
3.2	Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.	3
3.3	★ Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.	3

California Grade Seven Mathematics Standards

3.4	★ Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.	3
3.5	Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.	7
3.6	★ Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).	7

California Standard

Statistics, Data Analysis, and Probability

1.0	Students collect, organize, and represent data sets that have one or more variables and identify relationships among variables within a data set by hand and through the use of an electronic spreadsheet software program:	6
1.1	Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.	6
1.2	Represent two numerical variables on a scatterplot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).	6
1.3	★ Understand the meaning of, and be able to compute, the minimum, the lower quartile, the median, the upper quartile, and the maximum of a data set.	6

California Standard

Mathematical Reasoning

1.0	Students make decisions about how to approach problems:
1.1	Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.
1.2	Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.
1.3	Determine when and how to break a problem into simpler parts.
2.0	Students use strategies, skills, and concepts in finding solutions:
2.1	Use estimation to verify the reasonableness of calculated results.
2.2	Apply strategies and results from simpler problems to more complex problems.
2.3	Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.
2.4	Make and test conjectures by using both inductive and deductive reasoning.
2.5	Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.
2.6	Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.
2.7	Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.
2.8	Make precise calculations and check the validity of the results from the context of the problem.
3.0	Students determine a solution is complete and move beyond a particular problem by generalizing to other situations:
3.1	Evaluate the reasonableness of the solution in the context of the original situation.
3.2	Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.
3.3	Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.

Illustrated
throughout
Program

Advice for Parents and Guardians

What the Homework Book is For

Homework helps students improve their thinking skills and develop learning outside the classroom.

This Homework Book is an integral part of the CGP California Standards-Driven Course Two Mathematics Program.

- It focuses purely on the California Mathematics Content Standards for grade 7, with no extraneous content.
- It has been written to match the California Grade 7 Content Standards, using the Mathematics Framework for California Public Schools (2005) as a guide. The Standards are listed on pages iv–vi of this book.
- It has a clear and simple structure, which is the same for each component of the Program.
- It is a flexible Program, which caters for a diverse student body.

This Homework Book matches exactly the structure and order of teaching in the Student Textbook.

- The Course Two Program is broken down into eight Chapters (see pages ix–xi for more detail).
- These Chapters are in turn divided into smaller Sections, which cover broad areas of the Course Two Program.
- These Sections are then broken down into smaller, manageable Lessons, which are designed to be worked through in a typical 50-minute math class.
- Each Lesson in the Student Textbook starts with the relevant California Standard in full. This is followed by a clear learning objective written in everyday language, so that your child can understand what the Lesson is about and how it fits in with the overall California Grade 7 Standards.

At the end of each Lesson, the teacher will assign homework from this book, which contains one worksheet for each Lesson of the Course Two Program. That means that there is always additional work for your child to practice the skills learned in the math Lesson.

Each worksheet is perforated and hole-punched — so your child can hand each sheet in to the teacher, and then store the corrected work in a separate file at home or at school.

Using the Homework Book

The worksheets in this book have been designed to be straightforward to use.

The worksheets have many common features:

The relevant California Mathematics Standard is always stated at the start of each homework sheet.

Worked examples help students to answer problems and show them how to write their own solutions.

The pages are perforated so that each homework sheet can be pulled out and handed in to the teacher.

Name: _____ Class: _____ Date: _____

Lesson 2.4.4 **Problems Involving Fractions and Decimals**

California Standards: Number Sense 1.2, Mathematical Reasoning 2.2

Example

The length of a rectangular game table is $10\frac{1}{2}$ feet, while its width is 4.2 feet. What is the area of the game table?

Solution

Area = length \times width

$\rightarrow 10\frac{1}{2} \times 4.2$

$\rightarrow \frac{22}{2} \times 4.2$

$\rightarrow (4.2 \div 2) \times 21$

$\rightarrow 2.1 \times 21$

$\rightarrow 44.1$

The game table has an area of 44.1 ft².

Remember to include the correct units in your answer.

1. The width of a rectangular box is 10.3 inches while its length is $23\frac{1}{2}$ inches. Find the area of the box. _____

2. A rectangular driveway is $4\frac{1}{3}$ yards wide and 6.5 yards long. Determine the area of the driveway. _____

3. Find the area of the rectangle in the diagram.

Diagram: A rectangle with a width of 5 feet and a height of 7.5 feet.

4. A piece of fabric 12.75 feet long needs to be divided into shorter lengths of $1\frac{1}{2}$ feet long. How many full lengths can be made from this piece of fabric? _____

CGP Education Mathematics: Course 2 — Homework Book Lesson 2.4.4 61

There's one homework sheet for every Lesson in the book — so it's easy to refer back to the relevant part of the Textbook.

Useful hints to help students.

Each homework sheet covers several difficulty levels. The teacher has information about which problems are suitable for each student, so will set specific problems for your child.

At the back of the book you will find worked solutions to several problems from each worksheet. Together with the worked example on the worksheet itself, this will allow you to check that your child understands the key concepts.

This book contains more than just problems. Pages ix–xi give Chapter Overviews with lists of quick-reference Key Notation and Terminology, and pages xii–xiii contain advice on Question Technique.

Advice for Parents and Guardians

By getting actively involved in your child's education, you can make a real difference to his or her success.

Even if you are less confident with the math yourself, you can still provide help — both on a practical level and in less direct ways. Just by showing an interest in your child's work, you can help improve their perception of the value of math.

Here is some practical advice on how to help your child get the most out of their Course Two program.

Provide a Suitable Working Environment

It is important that your child has an appropriate place to work in. It is very easy to get distracted at home, so each of the measures suggested below is designed to keep your child as focused as possible.

- He or she should work at an uncluttered table or desk — a kitchen table is fine for this.
- The environment should contain as few distractions as possible — for example, if there is a TV in the room, make sure it is turned off and your child is facing away from it (even a turned off TV can be distracting).
- Try to set aside a regular time each day for homework. This can be difficult to fit around other commitments, but it is worth making the effort. Having a homework slot as part of your family's regular schedule can help your child to get into the right frame of mind to work.
- Encourage your child to work solidly for twenty minutes, then take a five minute break. Even with the best intentions, many children find it hard to focus for long periods of time — so working in short, intensive bursts is often most effective. The short break can also be used as an incentive — working hard for twenty minutes straight earns five minutes off.

Try to Identify Problem Areas

If your child is struggling with a piece of work, there are usually several possible root causes. Try to identify which aspects of a question your child is having difficulty with, so that he or she can take steps to solve the problem.

- Ask your child to explain in words how the problem would be solved. Struggling with this may suggest that the basic concept behind the question has not been grasped. (You do not have to understand the math yourself to do this, you just need to judge whether your child can explain the concept clearly.) See page xii, Concept Questions. It can be useful to go through this process even if your child is doing well. Students often learn procedures for answering questions without understanding the underlying math. Although this can be sufficient for simple problems, they may run into difficulties later if important concepts aren't fully understood.
- If your child understands the concept, but still cannot answer the question correctly, the problem may be with a particular component skill (for example, one step in the work). Read through a worked example with your child (you will find these in every Lesson of the Homework Book). Then copy out the question onto a separate piece of paper and ask your child to try to answer it. Compare each step of the work with that given in the book to see where the problem is.
- The difficulty may lie with the type of question. On pages xii and xiii, Guidance on Question Technique, there are lists of measures that can be taken to deal with particular question types.

Keep in Contact with the School

You ought to receive regular reports on your child's progress from the school, but remember that communication between home and school can be in two directions. If you are concerned about any aspect of your child's progress, it may help to discuss the issue with his or her teacher.

Overviews of Chapter Content

Course Two is not a course that can be treated in isolation. In each Chapter, you learn new concepts that are part of the larger picture of mathematics, and everything you learn builds on your knowledge from previous grades.

Chapter One — The Basics of Algebra

Chapter 1 is about expressions, equations, and inequalities.

The following concepts are covered in this Chapter:

- using variables to represent unknown quantities
- writing expressions, equations and inequalities using variables
- evaluating expressions by substituting numbers for variables
- simplifying expressions by collecting like terms, and by using math properties
- the correct order in which to carry out mathematical operations
- solving two-step equations, and using them to solve real-life problems
- plotting the solutions to inequalities on a number line

How the Chapter follows on from previous study:

In grades 5 and 6, you wrote expressions using variables and evaluated them. In grade 6, you wrote and solved one-step equations, and also used the correct order of operations to solve problems.

Key Notation and Terminology:

Properties of Numbers

associative properties:	$(x + y) + z = x + (y + z)$ $(xy)z = x(yz)$
commutative properties:	$x + y = y + x$ $xy = yx$
distributive property:	$x(y + z) = xy + xz$
identity properties:	$a + 0 = a$ $a \cdot 1 = a$
inverse properties:	$a + -a = 0$ $a \cdot \frac{1}{a} = 1$

Order of Operations

PEMDAS: shows which parts of an expression should be evaluated first;
Parentheses, Exponents, Multiplication/Division,
Addition/Subtraction

Algebraic Terminology

expression:	a collection of numbers, variables, and symbols that represent a quantity
equation:	a mathematical statement showing that two quantities are equal
inequality:	a mathematical statement showing how two quantities compare to each other

Chapter Two — Rational and Irrational Numbers

Chapter 2 is about identifying rational numbers and calculating with them.

The following concepts are covered in this Chapter:

- what rational numbers and irrational numbers are
- converting decimals to fractions
- finding the absolute values of numbers and expressions
- adding, subtracting, multiplying, and dividing integers, fractions, and decimals
- finding powers of integers, fractions, and decimals
- finding roots of perfect square numbers, and estimating roots of nonperfect square numbers.

How the Chapter follows on from previous study:

Since early grades, you have been computing with whole numbers to solve problems. In grade 4, you learned standard methods for adding, subtracting, multiplying, and dividing multidigit numbers. In grade 3, you started doing simple calculations with decimals and fractions, and you developed these skills further through grades 5 and 6.

Key Notation and Terminology:

Working with Fractions

denominator:	the bottom line of a fraction
numerator:	the top line of a fraction
mixed number:	a number with a whole number part and a fraction part (like $2\frac{1}{2}$)
reciprocal:	the number you must multiply something by to get 1. To find the reciprocal of a fraction, you swap over the numerator and denominator

Absolute Value

The absolute value of a number is its distance from 0 on the number line.

For example, $|-5|$ means the absolute value of -5 .
 $|-5| = 5$, and $|5| = 5$

Rational Numbers

A rational number is a number that can be written as $\frac{a}{b}$, where both a and b are integers (and b is not equal to 0)

Overviews of Chapter Content

Chapter Three — Two-Dimensional Figures

Chapter 3 is about two-dimensional shapes and the coordinate plane.

The following concepts are covered in this Chapter:

- finding the perimeter and area of a range of shapes
- plotting points and drawing shapes on the coordinate plane
- using the Pythagorean theorem to find lengths in right triangles
- translating and reflecting shapes
- using scale factors to resize shapes and produce scale drawings
- constructing geometrical features, such as perpendicular and angle bisectors

How the Chapter follows on from previous study:

In grade 4, you used formulas to find the areas and perimeters of rectangles, and plotted points on the coordinate plane.

In grade 5, you found areas of parallelograms, and in grade 6, the circumferences and areas of circles.

Key Notation and Terminology:

Geometry

similar: two figures are similar if they are the same shape (they don't have to be the same size)
congruent: exactly the same size and shape

Circles

radius: the distance from the center of a circle to its edge

diameter: the distance from one side of a circle to the other, passing through the center

Other Shapes

parallelogram: a four-sided shape with two pairs of parallel sides

rhombus: a parallelogram with four equal-length sides

trapezoid: a four-sided shape with exactly one pair of parallel sides

Chapter Four — Linear Functions

Chapter 4 is about graphing linear equations, rates, converting units, and solving inequalities.

The following concepts are covered in this Chapter:

- graphing linear equations and finding their slopes
- solving problems involving rates, including speed
- plotting quantities from rates on graphs
- converting units within and between measurement systems
- solving inequalities

How the Chapter follows on from previous study:

In grade 6, you learned what rates are, and used proportions to solve problems. You also converted between units of measure. In grade 5, you graphed ordered pairs resulting from an equation. Solving inequalities follows on from writing inequalities in Chapter 1.

Key Notation and Terminology:

Ratios and Rates

ratios: these are ways to compare two quantities
rate: a special kind of ratio that has units
proportions: equations showing that two ratios are equal

Linear Equations and their Graphs

linear equations: these can be expressed in the form

$$y = mx + b$$

slope: a measure of the steepness of a line.

Slope = change in y ÷ change in x

Measurement Systems

customary units: units like inches, feet, miles, and pounds

metric units: units like meters, kilometers, and kilograms

Chapter Five — Powers

Chapter 5 is about positive and negative powers, monomials, and graphing nonlinear functions.

The following concepts are covered in this Chapter:

- multiplying and dividing powers with a common base
- working with negative and zero exponents
- multiplying and dividing monomials
- raising monomials to powers, and finding their square roots
- graphing functions of the forms $y = nx^2$ and $y = nx^3$

How the Chapter follows on from previous study:

In Chapter 2, you evaluated powers of rational numbers, and found square roots. In Chapter 4, you graphed linear functions.

Key Notation and Terminology:

Powers

multiplication of powers rule: $a^m \cdot a^n = a^{(m+n)}$

division of powers rule: $a^m \div a^n = a^{(m-n)}$

zero exponent rule: for any number $a \neq 0$,
 $a^0 = 1$

negative exponents: for any number $a \neq 0$,
 $a^{-n} = \frac{1}{a^n}$

raising a power to a power: $(a^m)^n = a^{m \cdot n}$

Monomials

monomial: an expression with only one term

coefficient: the number that the variables in a monomial are multiplied by

Overviews of Chapter Content

Chapter Six — The Basics of Statistics

Chapter 6 is about organizing, representing and analyzing data sets.

The following concepts are covered in this Chapter:

- calculating the median and the range
- creating and interpreting box-and-whisker and stem-and-leaf plots
- creating and interpreting scatterplots

How the Chapter follows on from previous study:

Since the early grades you have been representing data sets in a variety of forms, such as bar graphs. In grades 4, 5, and 6, you looked at the mean, median and mode. In grade 6, you analyzed data displays and considered different methods of collecting data.

Key Notation and Terminology:

Statistics

median:	the middle value in an ordered data set
range:	the difference between the greatest and least values in a data set
quartiles:	these split an ordered data set into four equal groups
lower quartile:	the median of the first half of an ordered data set
upper quartile:	the median of the second half of an ordered data set

Chapter Seven — Three-Dimensional Geometry

Chapter 7 is about attributes of solid shapes. Lines and planes in space are also discussed.

The following concepts are covered in this Chapter:

- identifying three-dimensional shapes
- creating nets of three-dimensional shapes
- finding surface areas and volumes of shapes
- graphing volumes of shapes against edge lengths
- how lines and planes can be arranged in space
- resizing solid shapes using a scale factor

How the Chapter follows on from previous study:

In Chapter 3, you found the areas of two-dimensional shapes and applied scale factors to them. In grade 5 you constructed rectangular prisms from nets, and found their surface areas and volumes. Then in grade 6, you used formulas to find the volumes of triangular prisms and cylinders.

Key Notation and Terminology:

Three-Dimensional Figures

prism:	a 3-D shape formed by joining with straight edges two congruent, parallel polygon faces
cylinder:	a 3-D shape that is like a prism, except that its bases are circles or ellipses
pyramid:	a 3-D shape that has a polygon for its base, and all the other faces come to a point
cone:	a 3-D shape that is like a pyramid, except that its base is a circle or ellipse

Lines and Planes in Space

coplanar lines:	lines that lie on the same plane
skew lines:	lines that neither intersect nor are parallel
perpendicular planes:	planes that intersect at right angles

Chapter Eight — Proportional Reasoning and Percents

Chapter 8 is about percents and percent change, rounding, and estimation.

The following concepts are covered in this Chapter:

- changing fractions and decimals to percents
- using percent increases and decreases to solve real-life problems
- rounding answers sensibly, and checking answers using estimation

How the Chapter follows on from previous study:

In grades 5 and 6, you calculated percents, and in grade 6, you solved problems involving discounts, simple interest, and tips.

Since grade 4, you have rounded numbers, and used estimation to check whether answers are reasonable.

Key Notation and Terminology:

Percents

$$1\% = 0.01 = \frac{1}{100}$$

Interest

simple interest: $I = Prt$
(where I is the interest earned, P is the principal investment, r is the yearly percent interest rate written as a fraction or decimal, and t is the time of the investment in years)

compound interest: $A = P(1 + rt)^n$
(where A is the amount in the account, P is the principal investment, r is the yearly percent interest rate written as a fraction or decimal, t is the time between each interest payment in years, and n is the number of interest payments made)

Guidance on Question Technique

There are a number of different abilities required in order to be successful in Course Two — concepts have to be understood, skills have to be learned, and those skills should be applied to new situations. So math questions are not all the same.

This Course uses a Variety of Question Types

The five main types of question covered are:

- **Concept Questions**
- **Skills Practice Questions**
- **Applying Skills to New Situations**
- **Interesting/Challenging Questions**
- **Proof Questions**

In addition to this, many questions will be a mixture of types — for example, an application of learned skills to a new situation may also be an interesting mathematical problem.

It's Important to Understand These Differences

All of these types of questions can be tackled in different ways. If you struggle with one particular type, there are specific measures you can take, as outlined below.

Concept Questions

Concept Questions are those that help you to understand the topic, and help to check your understanding.

These may be probing questions asked by your teacher as part of the teaching process, such as: **“Dividing by $\frac{1}{2}$ is the same as multiplying by which number? Why?”**

Or they may be exercises that reinforce and check understanding, such as: **“State whether each of the shapes a) – f) is a pyramid, or a circular cone.”**

Concept Questions are fundamental to the learning of mathematics, as they are based on understanding rather than skills. If you do not understand a concept, you may struggle to learn the necessary skills. At the same time, if you have a firm grasp of the concepts, the skills you learn will make much more sense.

If you are struggling with Concept Questions:

- 1) Go back to earlier work that you are comfortable with. This will give you a useful starting point.
- 2) Then gradually work through the concepts, one by one. Try looking at worked examples, making sure that you follow the reasoning behind every step.
- 3) This will give you a better understanding of where the math comes from than if you had merely “rote-learned” the facts.
- 4) If you can, return to the original questions that you were struggling with to check that you now understand the concepts.

Skills Practice Questions

These are drill-type questions that let you practice the skills you have just learned, and check that you have learned those skills properly, for example, **“For each of parts a) to v), use long division to find the quotient.”**

Skills Practice Questions are repetitive, and are designed this way to help you learn — it is generally easier to remember something that you’ve done 10 times, than something you’ve only done once.

These questions do not go beyond the scope of what you have learned in class; they simply provide lots of practice at using the same skills, over and over again.

If you are making mistakes in Skills Practice Questions, there are a few possible root problems:

- 1) It could be that you have not fully understood the concept, and so are not applying the method correctly. In this case, see the advice for Concept Questions.
- 2) If you do understand the concept, it may simply be that you need to brush up on one or more of the component skills. Try doing a worked example and comparing each of your steps with the steps in the book. That way you can see exactly where you are going wrong and get extra help with those topics if necessary.
- 3) To check that you have learned the skills required, practice them again and again until you consistently perform well. There are plenty of questions in this book and in the Textbook, and your teacher may be able to provide you with extra questions.

Problem Solving — Applying Skills to New Situations

These kinds of questions give you more practice at using learned skills, but they also require problem-solving abilities. They are often real-life applications of theoretical problems, for example, **“A distribution company sent out five times more crates of oranges on Tuesday than it did on Monday. Over the two days, it sent out 120 crates of oranges. How many crates of oranges did it send out on Tuesday?”** You need two distinct abilities here: the skills required to solve the equations, and the ability to translate the real-life problem into math. If you are struggling with this type of question, it is very important to pinpoint the cause of the difficulty.

- 1) If you have difficulty even starting these questions, then you need practice at problem solving, and translating real-world problems into math. Try to get your teacher (or other students) to work through some real-world examples with you. Ask them to start with simple examples, and move on only when you have understood each one.
- 2) If you can translate the problem into math, but then you solve it incorrectly, you may need to review and relearn the necessary skills. Have a look at the advice on Skills Practice Questions.
- 3) You may be able to solve real-world problems “in your head,” without the need to write down your method. While this is an equally valid way of solving the problem, you should realize that it is important to explain all your steps — if only for the purposes of checking any mistakes later on. If you find this difficult, you could start by explaining your reasoning to someone, and ask them to help you to write that “in math.”

Not all Applying Skills Questions will be real-life applications. Some will be Challenge Questions (see below), where you will be asked to apply your skills to different kinds of theoretical problems.

Interesting/Challenging Questions

These are questions designed to interest or challenge you, particularly once you have already mastered the basics. They are usually Applying Skills Questions, but are generally more difficult and often involve several different skills in one question, for example, **“Convert 20 miles per hour into meters per second.”**

This is a conversion between units, but here compound units of “meters per second” and “miles per hour” are used, whereas you may have only so far have converted between simpler units, such as “meters” to “miles,” or “seconds” to “hours.” The skills are the same, but you need to figure out how to apply those skills to a more difficult problem.

These questions are designed to stretch and challenge you, and are the kind that will usually only be set in class, where you can get help from your teacher or other students to work through the problem. In these questions it is not always the math that is more difficult — the questions sometimes involve different ways of thinking. So, even if you struggle with some areas of math, you may still be able to make good headway with some of the Challenge Questions. The most important thing is to not get fazed by them — try applying what you know and see what happens.

Proof Questions

Some questions ask you to use logical arguments to show that a mathematical statement is true or false, for example, **“Show that the diagonals of a rectangle are congruent.”**

Many people struggle with this idea, and are more comfortable with questions where you need to “find an answer.” However, the processes are the same whether you are finding an answer or proving that a given answer is true. Each step should be justifiable, or it may be incorrect.

It is helpful to have a list of useful properties and formulas next to you when you are trying to prove something. If you find yourself unsure of the next step, look down through the list and see if you can apply any of them. If you find something that you think that you can apply, try it and see what happens. If you struggle with this, try to get your teacher (or other students) to work through plenty of examples with you.



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Lesson

1.1.1

Variables and Expressions

CA Standards covered: Algebra and Functions 1.1, 1.4

1. Circle each variable expression.

a. t

b. $-5 + w$

c. 111

d. $0v$

e. $8(3 - a)$

2. Why is 45,000 not a variable expression?

3. Circle each numeric expression.

a. $h + 23$

b. $-2 - 12$

c. 25×3

d. $8 + v$

e. $155 \div k$

4. Say how many terms are in each of the expressions below.

a. 12×9

b. $12g + 14b$

c. $g + 11b$

d. $abc + def + ghi$

5. Write variable expressions that mean the same as each of the following word expressions.

a. z is multiplied by threeb. q less than 15c. one half g d. b more than 23

6. Write word expressions that mean the same as each of the following variable expressions.

a. $4m$

b. $x - 26$

c. $6(p + 34)$

d. $\frac{w}{8} - 125$

Example

Evaluate each expression given that $r = -1$, $t = 3$, and $u = 7$.

- a. rtu
- b. $4t - u$
- c. $t(u + r)$

Solution

- a. $rtu = (-1)(3)(7) = -21$
- b. $4t - u = 4(3) - 7 = 12 - 7 = 5$
- c. $t(u + r) = 3[7 + (-1)] = 3(6) = 18$

7. Evaluate each expression given that $k = 6$ and $m = 2$.

- | | | | |
|------------|-------|-----------------|-------|
| a. $k + 2$ | | b. $m \times 3$ | |
| c. $k + m$ | | d. $k - m$ | |

8. If $v = 1$, $w = -4$, $x = 0$, $y = 3$, and $z = -6$ then evaluate each expression.

- | | | | |
|-----------------|-------|------------------------|-------|
| a. $v + y$ | | b. $4w$ | |
| c. $2y + x - v$ | | d. wxy | |
| e. $(v - y)z$ | | f. $v + 5x - y(v - w)$ | |

9. If $v = 1$, $w = -4$, $x = 0$, $y = 3$, and $z = -6$ then evaluate each expression.

- a. The sum of w and z
- b. Double the sum of y and v
- c. z divided by y

Lesson
1.1.2

Simplifying Expressions

CA Standards covered: Algebra and Functions 1.3, 1.4

Like terms have the same variable, like $8y$ and $3y$,
or are both constants like 25 and 17.

1. Simplify the following expressions by collecting like terms.

a. $x + 2 + x$

b. $x + x + x + 4 + 8 + x$

c. $2x - 9 + x$

2. Simplify the following expressions by collecting like terms.

a. $6x - 4y + 3y$

b. $4x + 15 + 2x - x + 9$

c. $j - k - j + 3k$

3. Simplify the following expressions by collecting like terms where possible.

a. $5r + 3r + 4rt$

b. $2mn + 3m$

c. $7yz + yz - x$

4. Evaluate the following expressions using the distributive property.

a. $12 \times (6 + 2)$

b. $-5 \times (3 + 7)$

c. $2.5 \times (8 - 1)$

5. Write two different formulas for the area of the rectangle shown below — one with parentheses and one without.



and

6. Simplify each of the following expressions using the distributive property.

- a. $5(x + y)$ b. $-3(5 + t)$ c. $(r - 3e)14$ d. $(5 - w)9$ e. $-6(-9 + p)$

7. Fill in the missing part of each problem to make the statements true.

- a. $6(h + \text{-----}) = 6h + 66$ b. $8(g + \text{-----}) = 8g + 32h$
c. $4(\text{-----} - 12) = 4m - 48$ d. $\text{-----}(7 + 9u) = -14 - 18u$
e. $(15 - 3t)\text{-----} = 45 - 9t$ f. $(2x + \text{-----})11 = 22x + 110$
g. $(19m + \text{-----})4 = 76m + 4$ h. $\text{-----}(5t - 4m) = 0$

8. Simplify each expression using the distributive property and combine like terms.

- a. $4(x + 10) + 3(k - 8)$ -----
b. $9(c + t) + 7(4t - c)$ -----
c. $8(2a - 1) + (r + 10t)1 + 0(a + 17t)$ -----

9. Andres is on the tennis team. Last year he bought 7 tubes of tennis balls. The year before he bought k tubes. Each tube holds three balls.

- a. Write an expression using parentheses to describe how many tennis balls Andres bought in the last two years.

- b. Given that $k = 6$, evaluate your expression using the distributive property. -----

10. Shelly charges \$10 per hour for her babysitting services. Last month Shelly babysat for 15 hours. This month she babysat for n hours.

- a. Write an expression using parentheses to describe how much money she made. -----

- b. Given that $n = 12$, evaluate your expression using the distributive property. -----

Lesson
1.1.3

The Order of Operations

CA Standards covered: Algebra and Functions 1.2

Example

What is $5 + 4^2 \div 2 \times 8$?

Solution

$$\begin{aligned} &5 + 4^2 \div 2 \times 8 \\ &= 5 + 16 \div 2 \times 8 \\ &= 5 + 8 \times 8 \\ &= 5 + 64 \\ &= 69 \end{aligned}$$



One way to remember the order of operations is to remember the word "PEMDAS" — do parentheses first, then exponents, then multiplication and division, and finally addition and subtraction.

1. What is $144 + 3 \times 4 - 6$?

.....

2. Simplify the following expressions as far as possible.

a. $m \times (6 + 3) + 10$ b. $4 + b \times 7 - 2$

3. Two students evaluated $70 - 2(5 + 3)$. Their work is shown below. Who has the correct answer?

Student A

$$\begin{aligned} &70 - 2(5 + 3) \\ &70 - 2(8) \\ &68(8) \\ &544 \end{aligned}$$

Student B

$$\begin{aligned} &70 - 2(5 + 3) \\ &70 - 2(8) \\ &70 - 16 \\ &54 \end{aligned}$$

.....

4. Simplify each of the following expressions using the order of operations:

a. $14 \times 2 - 10$

.....

b. $12 \div 3 + 6$

.....

c. $5 \times 6 + 8 \div 2 - 4(2 \times 3)$

.....

d. $4^2 + 5 - 2 + (-1)^3$

.....

e. $[(4 \times 3) \div 6] + (5^2 - 3) \div 2 + 2$

.....

$(-1)^3 = (-1)(-1)(-1) = -1$



5. Evaluate each of the following expressions.

a. $(5 \times 4)^2$

b. $2[13 - (1 + 6)]$

c. $48 \div 3 + 5$

6. Insert parentheses into $20 + 9 - 8 \times 2$ to make it equal to 22.

7. A roof repairer works out that he will need to work with a colleague to fix a homeowner's roof. The roof repairer charges \$45 per hour and the colleague charges \$20 per hour. In the end, they took 6 hours to fix the roof.

a. Write an expression with parentheses to describe how much the homeowner owes for labor.

.....

b. Evaluate your expression to calculate the cost of the labor.

.....

8. Add grouping symbols to the expression $2 + 6 - 3 \times 4$ so that it equals 20.

9. Judy bought 6 CDs priced at \$12 each. She paid for \$10 of the price, then split the rest between five of her friends.

a. Write an expression to describe how much each of her friends paid.

b. Evaluate your expression to calculate how much each friend paid.

10. Simplify the expression $w \div (3 + 2 \cdot 5)$ as far as possible.

11. Simplify the expression $2k + (3 - 2)m + 16k \div 4$ as far as possible.

Lesson

1.1.4

The Identity and Inverse Properties

CA Standards covered: Algebra and Functions 1.3

1. a. What number is the additive identity?
- b. What number is the multiplicative identity?
2. a. What is the additive inverse of 17?
- b. What is the multiplicative inverse of 17?
- c. What is the additive inverse of -6.2 ?
- d. What is the multiplicative inverse of -6.2 ?
3. Complete each of the following expressions.
 - a. $x + \dots = x$
 - b. $j \times \dots = j$
 - c. $\dots + 0 = m$
4. Give the additive inverse of each of the following numbers.
 - a. 2
 - b. $\frac{1}{5}$
 - c. -250
5. Give the multiplicative inverse of each of the following numbers.
 - a. $\frac{3}{5}$
 - b. -12
 - c. $-\frac{7}{10}$
6. Complete the table below.

Number	Additive Inverse	Multiplicative Inverse
$\frac{2}{3}$		
-7		
$3\frac{2}{5}$		
$\frac{1}{5}$		

First write the number as an improper fraction

7. Use inverse operations to solve the following equations.

a. $s - 7 = 14$

b. $a + 4 = 12$

c. $9m - 4 = 32$

Example

Simplify $8(x + 2) - 16$. Justify your work.

Solution

$$\begin{aligned} & 8(x + 2) - 16 && \text{distributive property} \\ & = 8 \cdot x + 8 \cdot 2 - 16 && \text{the inverse property of addition} \\ & = 8x + 16 - 16 \\ & = 8x + 0 && \text{the identity property of addition} \\ & = 8x \end{aligned}$$

8. Simplify the expression $3(5 - \frac{1}{3}y) - 15$. Justify your work.

.....

.....

.....

.....

9. Sue thinks that all numbers have a multiplicative inverse, but Gabe disagrees. Who is right, and why?

.....

.....

10. Simplify the expression $15(x + 2) - 30 + x$. Justify your work.

.....

.....

.....

.....

Lesson

1.1.5 The Associative and Commutative Properties

CA Standards covered: Algebra and Functions 1.3

1. Is there a value of x for which $(x - 4)$ and $(4 - x)$ are the same?

2.
 - a. Use the associative property of addition to simplify $4p + (2p + 3)$
 - b. Use the associative property of addition to simplify $67 + (5 + n)$
 - c. Use the associative property of multiplication to simplify $-4(6m)$
 - d. Use the associative property of multiplication to simplify $8 \times (10 \times z)$
 - e. Use the commutative property of addition to simplify $12 + n + 2 + n$
 - f. Use the commutative property of multiplication to simplify $5 \times m \times 3$
 - g. Use the commutative property of multiplication to simplify $-4 \times a \times 6 \times -b$

3. Which property says that $(xy)z = x(yz)$?

4. Which property says that $x + (y + 3) = x + (3 + y)$?

5. Lorenzo says that the definition of subtraction and the commutative property can be used to show that $a - b$ is the same as $-b + a$. Is he right? Explain your answer.
.....
.....
.....

6. Which property says that $a + (b + c) = (a + b) + c$?

7. Fill in each of the blank spaces below to justify the action that has just been taken.

a. $[18 + (2 + 45)] \times 5 \times (18 - 16)$

$= [(18 + 2) + 45] \times 5 \times (18 - 16)$

$= 65 \times 5 \times 2$

$= 65 \times 2 \times 5$

$= 650$

b. $6 \times [12 \times (x + 4)] \times 1$

$= 6 \times 1 \times [12 \times (x + 4)]$

$= 6 \times [12 \times (x + 4)]$

$= (6 \times 12) \times (x + 4)$

$= 72(x + 4)$

8. Simplify $[(8x + 23) + 9] \times 3$. Justify your work.

.....

.....

.....

.....

9. Fill in each of the blank spaces below to justify the action that has just been taken.

$[(12 + x + 3) \times 8] \times 9$

$= [(12 + 3 + x) \times 8] \times 9$

$= [(15 + x) \times 8] \times 9$

$= (15 + x) \times (8 \times 9)$

$= (15 + x) \times 72$

$= 72(15 + x)$

Lesson
1.2.1

Writing Expressions

CA Standards covered: Algebra and Functions 1.1

Example

Translate each of the following sentences into variable expressions.

- a. 452 less than a number, x .
b. Forty more than 22 divided by a number, x .

Solution


- a. $x - 452$.
b. $40 + \frac{22}{x}$, or $\frac{22}{x} + 40$.

1. Match each of the following sentences with the variable expression that means the same thing.

- | | | | |
|--------------------------------------|-------|-------------------|-------------------|
| a. 27 decreased by a number, x . | | A. $\frac{x}{27}$ | B. $\frac{27}{x}$ |
| b. A number, x , multiplied by 27. | | C. $x - 27$ | D. $27 - x$ |
| c. A number, x , divided by 27 | | E. $27 + x$ | F. $27x$ |
| d. 27 more than a number, x . | | | |

2. Translate each of the following sentences into a variable expression.

- a. 92 increased by a number, x
b. 45 divided by a number, x
c. 14 decreased by a number, x
d. A number, x , multiplied by one thousand.

 In expressions involving multiplication, you normally write the number before the variable — for example, $3k$ rather than $k3$.

3. Fill in the missing parts so that each sentence matches the variable expression on its right.

- | | | |
|--------------------------|--------------------------------------|----------------|
| a. Fifteen | a number, r . | $r - 15$ |
| b. 27 | a number, b . | $b + 27$ |
| c. A number, w , | 20. | $\frac{w}{20}$ |
| d. 35 | two different numbers, x and y . | $35xy$ |

4. Translate each of the following sentences into a variable expression.

a. Ten less than the product of a number, x , and 5.

b. Forty more than triple a number, x

c. Twelve decreased by a quarter of a number, x

5. Write variable expressions that could be evaluated to answer each of the following questions. Use x as the variable and say what it represents.

a. A parallelogram has a height of 5 cm. What is its area?

b. Esperanza has four marbles fewer than Rhonda. How many marbles does Esperanza have in total?

6. Circle the expression that means “eleven minus the product of 6 and a number.”

a. $6n - 11$ b. $(6 \div n) - 11$ c. $11 - 6n$ d. $11 - (5 \div n)$

7. Write a variable expression for each of the following situations. Use b to represent “a full box of pencils.”

a. A full box of pencils with 3 extra pencils.

b. Three full boxes of pencils

8. Wenda bought 12 more carnival ride tickets than Isabella, who bought i tickets. Write an expression for the number of carnival ride tickets Wenda bought.

9. The Stebiliskis had their water heater replaced by a plumber. The plumber charged \$85 per hour and the water heater cost \$525. Write a variable expression for the amount of money the Stebiliskis owed the plumber.

First state the unknown, “let h = the number of hours the plumber worked for.”

10. Match each of the following real-life examples to the appropriate variable expression.

a. Frieda has 45 less dollars than the amount Yolanda has, y

A. $45y$

b. Jorge has 45 divided by y movies.

B. $y - 45$

c. Renee has 45 CDs and Lauren has y . How many CDs do they have in total?

C. $y + 45$

d. Ashland is tutored for y hours a month and his tutor charges \$45 an hour.

D. $\frac{45}{y}$

What is the total amount in dollars due to the tutor at the end of the month?

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Lesson
1.2.2

Equations

CA Standards covered: Algebra and Functions 1.4

1. Identify each of the following as an expression or an equation.

a. $6w$

b. $12 = q - 8$

c. $29 = \frac{r}{4}$

d. $15 - t$

e. $3(t - 45) = 125$

2. Identify each of the following as a variable or numeric equation.

a. $s + 23 = 9$

b. $9 \div 3 = 3 \times 1$

c. $11 + d = e \div 9$

d. $24 + 23 = 19 + 28$

e. $19 \times 3 = 342 \div 6$

Example

On Monday, Omar received eight fewer instant messages than Pablo. In total Omar received 27 instant messages on Monday. Turn this situation into an equation.

Solution

Let p = number of instant messages Pablo received on Monday.

Then Omar's instant messages = Pablo's instant messages $- 8$, so the equation is $27 = p - 8$.

What information's missing? Use this to identify the variable.

3. Translate each of the following sentences into equations.

a. The sum of half a number, x , and 44 is 128.

b. A seventh of a number, x , increased by 19, is 29.

c. A number, x , increased by itself is 64.

d. 16 is twice the sum of a number, x , and 17.

4. Write an equation that means the same as the sentence, “The sum of sixteen and g is equal to seventy divided by seven.”

5. Elizabeth owns 10 pairs of socks. Christian owned twice as many pairs as Elizabeth, but then he got an extra two pairs for his birthday. Write an equation for s , the number of pairs of socks Christian owns.

6. Crystal normally spends twice as long on the bus to school as Francisco, but last week the bus was delayed for ten minutes just before Francisco got on. Write an equation for the amount of time Crystal was on the bus on the day it was delayed, using c for the number of minutes Crystal was on the bus and f for the number of minutes Francisco was on the bus.

7. Devin and Edward race in the 110 meters hurdles for their school. Devin’s best time is 16 seconds. Given that Devin’s best time is seven-tenths of Edward’s best time, write an equation for Edward’s best time.

8. The diameter of a circle is found by doubling the radius. Write a formula for the diameter of a circle, d , in terms of its radius, r .

9. The perimeter of a triangle is the sum of all three side lengths of the triangle. Two side lengths of a triangle are 27 centimeters and 38 centimeters. If the perimeter of the triangle is 84 centimeters then write an equation using l , the length of the third side.

10. The formula for the surface area of a cylinder is $A = 2\pi r(r + h)$. What is the formula for the surface area of a cylinder where $r = 5$ and $h = 2w$?

Lesson
1.2.3

Solving One-Step Equations

CA Standards covered: Algebra and Functions 4.1

1. $y + 11 = 28$. If eleven is subtracted from the left-hand side of the equation then what number needs to be subtracted from the right-hand side to balance the equation?

2. Identify each of the following statements as either true or false:

a. If $5r - 7 = 58$ then $r = 7$.

.....

b. If $4p + 8 = 12$ then $p = 1$.

.....

c. If $\frac{w}{8} - 3 = 31$ then $w = 272$.

.....

d. If $3\left(\frac{a}{4}\right) - \frac{(4+a)}{2} = 1$ then $a = 12$.

.....

3. Match each equation with its solution.

a. $y - 7 = 22$

A. $y = -14$

B. $y = 0$

b. $y + 3 = -11$

C. $y = 30$

D. $y = 29$

c. $-8 + y = 22$

E. $y = 4$

F. $y = 28$

d. $14 = y - 14$

G. $y = -8$

H. $y = 15$

4. Solve each of the following equations.

a. $m + 4 = 20$

.....

b. $h + 9 = 12$

.....

c. $r - 3 = 28$

.....

d. $t - 10 = 10$

.....

5. Solve the following equations:

a. $t + 4 = 4$

b. $y + 25 = 0$

c. $18 = r - 12$

d. $-15 = w + 12$

6. Explain the error made solving the following equation, and solve it correctly.

$14 = r + 6$

$20 = r$

$r =$ -----

7. Match each equation with its solution.

a. $-8x = 48$ -----

A. $x = 25$

B. $x = 1$

b. $\frac{72}{x} = 9$ -----

C. $x = -24$

D. $x = -18$

c. $\frac{1}{5}x = 5$ -----

E. $x = 18$

F. $x = 8$

d. $-\frac{x}{3} = 8$ -----

G. $x = -8$

H. $x = -6$

e. $-\frac{x}{6} = -3$ -----

I. $x = 6$

J. $x = 24$

8. Is multiplying both sides by 8 the correct way to solve $8x = 72$? Explain your answer.

9. Solve the equation $5v = 35$. -----

10. Solve each of the following equations.

a. $7g = 56$ -----

b. $3h = 3$ -----

c. $\frac{w}{6} = 12$ -----

d. $\frac{r}{10} = 10$ -----

Lesson
1.2.4

Solving Two-Step Equations

CA Standards covered: Algebra and Functions 4.1

Example

Solve the equation $9m + 17 = 35$.

Solution

$$9m + 17 = 35$$

$$9m + 17 - 17 = 35 - 17$$

$$9m = 18$$

$$9m \div 9 = 18 \div 9$$

$$m = 2$$

1. If $2g + 3 = 13$ then what is g ?

2. Solve each of the following equations.

a. $5x + 1 = 26$

b. $7y - 10 = 60$

3. Say whether each of the following equations has been solved correctly.

a. $3x + 7 = 31$ $x = 8$

b. $50 = -2m + 6$ $m = -25$

c. $9x + 11 = 92$ $x = -9$

4. Solve each of the following equations.

a. $8r + 3 = 11$

b. $19 = 5r + 9$

.....

.....

5. Solve each of the following equations.

a. $-10x + 12 = 92$

b. $47 = 5y + 12$

c. $22 = 4 + 6x$

.....

.....

.....

6. Find the error in the following solution.

$$\begin{aligned} -7x + 22 &= 85 \\ -7x + 22 - 22 &= 85 - 22 \\ -7x &= 63 \\ -7x \div 7 &= 63 \div 7 \\ x &= 9 \end{aligned}$$

7. Say in what order you would undo the operations in each of the following equations.

- a. $70 = 3 \times (4 + y)$ -----
- b. $42n - 7 = 13$ -----
- c. $z \div 10 + 3 = 14$ -----

8. Solve each of the following equations.

- a. $3(x + 2) = 9$ -----
- b. $(y + 6) \div 2 = 12$ -----
- c. $(w \times 3) \times 2 = 24$ -----

9. Solve each of the following equations.

- a. $(q - 1) \times 3 = 18$ -----
- b. $22.5 = 7.5(k - 5)$ -----
- c. $(r + 0.75) \div 2 = 3.125$ -----

10. Two different students solved the equation $(n + 3) \div 6 = 16$. Their work is shown below.
Which student is correct?

Student A

$$\begin{aligned} (n + 3) \div 6 &= 16 \\ (n + 3) \div 6 \times 6 &= 16 \times 6 \\ n + 3 &= 96 \\ n + 3 - 3 &= 96 - 3 \\ n &= 93 \end{aligned}$$

Student B

$$\begin{aligned} (n + 3) \div 6 &= 16 \\ (n + 3) - 3 \div 6 &= 16 - 3 \\ n \div 6 &= 13 \\ n \div 6 \times 6 &= 13 \times 6 \\ n &= 78 \end{aligned}$$

11. Solve each of the following equations.

- a. $0.25(g + 6) = 128$ -----
- b. $1.6 = 0.8(e - 5.4)$ -----
- c. $15 = 135 \div h$ -----

Lesson
1.2.5

More Two-Step Equations

CA Standards covered: Algebra and Functions 4.1

Example

Solve the equation $\frac{4}{5}f = 16$.

Solution

$$\frac{4}{5}f = 16$$

$$4f \div 5 = 16$$

$$4f = 80$$

$$f = 20$$

1. Solve $\frac{2}{5}q = 32$

2. Solve each of the following equations.

a. $-7r - 5 = -12$

.....

b. $\frac{1}{8}g + 1 = 99$

.....

3. Solve each of the following equations.

a. $-16 = -9x - 16$

.....

b. $-\frac{m}{10} + 17 = -41$

.....

c. $\frac{2}{3}x = 29$

.....

4. What error was made solving the following problem?

$$\frac{1}{9}x + 27 = 63$$

$$\frac{1}{9}x = 36$$

take 27 from both sides

$$x = 4$$

multiply both sides by $\frac{1}{9}$

.....

5. Solve each of the following equations.

a. $15 = \frac{5}{9}h$

b. $17 \times \frac{z}{3} = -51$

c. $(x \div 6) + 5 = 29$

6. Carmela wants to buy a digital camera that costs \$249. She has \$24 and is saving \$15 each week. Write an equation and solve it to find out how many weeks she needs to save for to buy the camera.

7. Solve $6x - 2 = 10$ and use substitution to check your answer. -----

8. Decide whether each of the following equations has been solved correctly.

a. $\frac{1}{7}x + 3 = 17$

$x = 28$

b. $-25 = -3x + 14$

$x = -13$

c. $4x - 9 = -17$

$x = -2$

9. Solve each of the following equations and then check your answer by substituting it back in.

a. $8 = b \div 3 + 6$

b. $6n - 4 = -2$

c. $\frac{3}{5}d = 12$

10. Solve each of the following equations and then check your answer by substituting it back in.

a. $12 \div r = 3$

b. $31 = 15 + 2w$

c. $(6 + g) \div 12 = -8$

Lesson
1.2.6

Applications of Equations

CA Standards covered: Algebra and Functions 4.1, Mathematical Reasoning 2.1

1. The Piersons have paid \$75 to enroll their son Fred at preschool. They will pay \$152 a month in fees. Write and solve an equation to work out how many months it will take for the total bill to reach \$683.

2. The formula for the area of a triangle is $\frac{1}{2}bh$, where b is the length of the base of the triangle and h is the vertical height. Use the formula to find the height of a triangle with a base length of 8 inches and an area of 36 square inches.

3. Quincy paid \$9.50 to join a photography club that allows him to print digital pictures for \$0.20 each. So far he has paid \$12.50 to the club. Write and solve an equation to work out how many pictures Quincy has paid for.

4. A coffee shop uses the formula $P = C + \frac{1}{2}S$ to calculate the number of loyalty points a customer has earned after buying C coffees and S sandwiches.

a. If Pete has 32 loyalty points after buying 23 coffees, then how many sandwiches has he bought?

b. If Morgan has 22 loyalty points after buying 14 sandwiches, then how many coffees has she bought?

5. Tamika is three inches taller than Mackenzie. If Tamika is 65 inches tall then how tall is Mackenzie? Set up and solve an equation to reach your answer.

6. The sum of the angles in a triangle is 180 degrees. In a particular triangle the first angle is 18 degrees less than the second angle, and the third angle is 70 degrees. Write and solve an equation to find the measures of the first two angles.

7. Rueben is six months old and naps twice a day. At 10 a.m. Rueben takes a 50 minute nap which is one-third the length of his 3 p.m. nap. Write and solve an equation to find the length of Reuben's 3 p.m. nap.

8. A school wants to investigate attendance at its recent middle school play.

Wednesday's audience was half as large as Thursday's.

On Thursday, 270 people saw the play.

Friday's audience was three times as large as Wednesday's.

On Saturday there were 60 fewer people in the audience than on Friday night.

Write and solve equations to find the number of people that attended each night, and find the total attendance across the four performances.

a. Wednesday: -----

b. Friday: -----

c. Saturday: -----

d. Total: -----

Lesson
1.2.7

Understanding Problems

CA Standards covered: Algebra and Functions 4.1, Mathematical Reasoning 1.1

Example

On Friday Cody swam 20 lengths. On Saturday he swam four lengths and on Sunday he swam twice as many lengths as on Thursday. Do you have enough information to work out how many lengths Cody swam from Friday to Sunday? If not then what extra information do you need?

Solution

You do not have enough information to answer the question. You would need to know how many lengths Cody swam on Thursday.

1. Monique runs a shoe store. Last month her profits were 50% more than in the month before. How much profit did she make last month?

Say what extra information, if any, is needed to solve this problem.

.....

2. A store has notebooks for sale at \$2 each. Nancy has a card that entitles her to a 25% discount at the store. If she buys three markers from the store how much will she have to pay?

Say what extra information, if any, is needed to solve this problem.

.....

3. A large office building buys 120 five-gallon bottles of spring water every month. A crate of bottles costs \$18.50. How much do they spend on water each month?

Say what extra information, if any, is needed to solve this problem.

.....

4. Determine what information, if any, is missing in each of the following questions.

a. A juice bar charges \$3 for an energy drink. Lori buys an energy drink and a banana. How much does Lori owe?

.....

b. Derrick has \$25 and is saving \$10 a week. How many weeks will he need to save to buy the bicycle he wants?

.....

c. An oak tree on the school campus is 2 feet shorter than half the height of a redwood tree. How tall is the oak tree?

.....

5. Jarome's mother started a crossword puzzle at 2 p.m. and finished 90 minutes later. Jarome's great aunt started a crossword puzzle at 5 p.m. and it took her twice as long as Jarome's mother plus 10 minutes to complete the puzzle. How long did it take Jarome's great aunt to finish the crossword puzzle?

Which pieces of information are not needed to answer the above problem?

6. The table on the right shows the prices a grocery store charges for different types of fruit and the size of bag they come in.

Type	Weight	Price
grapes	2 lb	\$3.50
apples	5 lb	\$4.00
bananas	3 lb	\$2.50
oranges	10 lb	\$8.00

- a. Lisa spent \$13 on 1 bag of oranges and some bananas.

How many pounds of bananas did she buy? -----

- b. Otto bought 1 bag of grapes and 3 bags of bananas.

How much did he spend? -----

- c. Ramundo bought 2 bags of apples and some grapes. He spent \$18.50.

How many pounds of grapes did he buy? -----

7. Three times the average depth of the Arctic Ocean is 114 meters more than 3000 meters. The average depth of the Earth's oceans is 3720 meters. Write and solve an equation to find the average depth of the Arctic Ocean and say which information was not relevant to the question.

8. Ralph rode his bike 45 miles in 3 hours. What are the correct units to state if you divide 45 by 3 to work out Ralph's speed? -----

9. Fill in the correct units for each of the following results.

- a. $5 \text{ persons} \times 15 \text{ hours} = 75$ ----- b. $70 \text{ elephants} \div 10 \text{ zoos} = 7$ -----

10. Fernando observed that his class usually drinks about 24 bottles of juice every 3 days. What units should he use to complete the calculation $24 \text{ bottles} \div 3 \text{ days} = 8$ ----- ?

11. Fill in the correct units for each of the following results.

- a. $\$30 \div 6 \text{ hours} = 5$ ----- b. $100 \text{ persons} \times 10 \text{ days} = 1000$ -----
- c. $2 \text{ kilowatts} \times 3 \text{ hours} = 6$ ----- d. $300 \text{ pounds} \div 100 \text{ square feet} = 3$ -----

Lesson

1.3.1

Inequalities

CA Standards covered: Algebra and Functions 1.4, 1.5

Example

Write a description of each of the following inequalities.

a. $v > 7$

b. $w \leq -11$

Solution

a. $v > 7$ means that v is greater than 7.

b. $w \leq -11$ means that w is less than or equal to -11 .

1. Identify each of the following as either an expression, an equation, or an inequality.

a. $5t = 25$

b. $t + 25$

.....

.....

c. $t < -5$

d. $w \geq 15$

.....

.....

2. Write a description of each of the following inequalities.

a. $t > 90$

.....

b. $r \leq -32$

.....

c. $18 > w$

.....

d. $0 \geq z$

.....

3. Translate each of the sentences below into an equation or an inequality.

a. w is 9

b. q is less than or equal to -27

c. v is greater than -12

d. 12 is p

e. 45 is greater than or equal to r

4. For each of the following inequalities, say whether its graph would have an open or closed circle.

a. $m \geq -3$

.....

b. $z > 4$

.....

c. $c < -17$

.....

d. $d \leq 2$

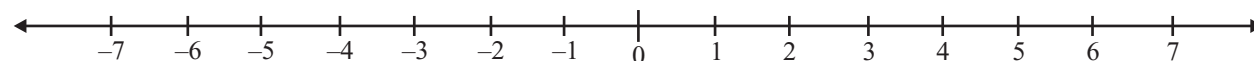
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e. $u \leq 22$

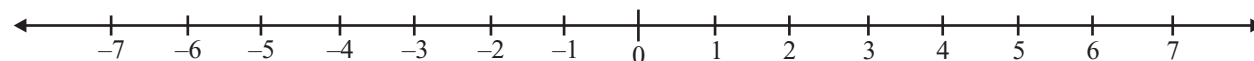
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5. Graph each of the following inequalities.

a. $w > 6$



b. $r \leq -7$



c. $h \geq 0$

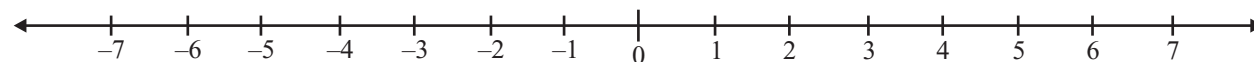


d. $t > -3$



6. The level of a river is never more than 4 meters above its normal height.

Graph an inequality that represents this situation.



Lesson
1.3.2

Writing Inequalities

CA Standards covered: Algebra and Functions 1.1

Example

Write an inequality to describe the sentence, “Six more than x is greater than or equal to fifty-two.”

Solution

“is greater than or equal to” is written with the symbol \geq .

The part before the “greater than or equal to” is “six more than x .” That’s written as $x + 6$.

The part after the “greater than or equal to” is “fifty-two.” So the full inequality is $x + 6 \geq 52$.

1. Complete each of the following inequalities by writing in the correct inequality sign.

- Four plus a number, x , is less than 15, so $4 + x$ ____ 15.
- A number, x , divided by 52 is greater than 9, so $x \div 52$ ____ 9.
- Sixteen is less than or equal to $23 - 5$, so 16 ____ $23 - 5$.
- Five times a number, x , is greater than or equal to 8, so $5x$ ____ 8.

2. Translate each of the following sentences into an inequality.

- The sum of 17 and a number, x , is greater than 29. _____
- The product of a number, x , and -9 is less than or equal to 28. _____
- Minus fifteen is less than a tenth of a number, x . _____
- Twelve less than a number, x , is greater than or equal to 45. _____

↖ If 25 is less than something else, then that something else is greater than 25.

3. Write “25 less than a number, x , is greater than 8” as an inequality in two different ways.

- _____
- _____

4. Fill in the missing words to match the inequality:

↖ It might help you to say the inequality out loud...

- $x + 3 < 5$ The sum of a number, x , and 3 is ____ 5.
- $42 \geq -6y$ Forty-two is ____ the product of -6 and a number, y .
- $x + y < 82$ The sum of two different numbers, x and y , is ____ 82.

5. For each inequality, circle the numbers that do not work when substituted for the variable.

a. $x - 2 < -6$ -11 -4 0 5

b. $4x \leq 0$ -6 -1 0 2

c. $\frac{x}{6} > 2$ -12 -6 0 12

6. Claire took \$68 on a shopping trip and ended up spending \$30 more than Hannah.
Write an inequality about the amount of money spent by Hannah.

.....

7. Kahlil has twice as many math problems for homework as Jarvis, but less than 18 problems in total.
Write an inequality about the number of math problems Jarvis has for homework.

.....

8. Patricia can jump rope half as many times as Oriana, but always manages to jump at least 19 times.
Write an inequality about the number of times Oriana can jump rope.

.....

9. Why does $x + 9 < 12$ mean the same as $12 > x + 9$?

.....

.....

10. Karina is 15. Her brother, Hector, is less than half her age. Write an inequality about Hector's age.

.....

11. Natalie takes three-quarters as long to get home after school as Justin, and never takes less than 50 minutes. Write an inequality about the amount of time it takes Justin to get home.

.....

Lesson
1.3.3

Two-Step Inequalities

CA Standards covered: Algebra and Functions 1.1

1. Translate each of the following sentences into inequalities.

a. The sum of half a number and 10 is greater than or equal to 58.

b. The sum of 4 multiplied by a number and 7 is less than -23

c. Seventeen is greater than or equal to the sum of one-third of a number and 43.

2. Translate each of the following inequalities into sentences.

a. $8x + 2 > 50$

.....

b. $15 - \frac{x}{2} \geq 12$

.....

c. $2x - 5 < 0$

.....

3. Complete each of the following inequalities using either $<$ or $>$ so that substituting the number in parentheses for x makes a true statement.

a. $2x - 8$ 22 (8)

b. $3x + 5$ 26 (-6)

c. -2 $-\frac{x}{3} + 5$ (24)

4. For each inequality, circle the numbers that do not work when substituted for the variable.

a. $2x - 5 < -1$ -4 -2 0 3 7

b. $0 \leq \frac{x}{3} + 6$ -36 -18 0 6 9

5. Given that $r + 5 \leq q$ and $q \leq 4t$, identify each of the following statements as true or false.

a. If $r = 8$ and $t = 12$, then q can be 8 b. If $r = -1$ and $t = -4$, then q can be 0

c. If $r = -10$ and $t = 3$, then q can be -1 d. If $r = 7$ and $t = 8$, then q can be 12

Example

A car wash company has a club for frequent customers. For \$35 a year membership, customers can get a complete inside and outside car wash for just \$13. Mr. Thomas wants to spend no more than \$320 on car washes for the year. Write an inequality to represent the number of car washes he can buy in that time if he joins the club.

Solution

Let x = the number of car washes Mr. Thomas buys. Then he will owe \$35 and another $13x$ dollars. He wants to spend no more than \$320, so he wants to spend less than or equal to \$320. That means that the inequality is $13x + 35 \leq 320$.

6. Every day, radio Station A plays 290 songs. Radio Station B plays at least 150 more than half the number of songs played on Radio Station A. Write an inequality about the number of songs played on Radio Station B.

.....

7. A florist places a minimum of 15 daisies in every arrangement. She also makes sure that the number of daisies in an arrangement is 7 more than twice the number of carnations. Write an inequality about the number of carnations in an arrangement.

.....

8. A storage unit requires a deposit of \$75 and has a monthly rental cost of \$90. Yesenia has \$800 to pay for the unit. Write an inequality for the number of months Yesenia can rent the storage unit.

.....

9. The Beard family are on vacation and plan to spend no more than \$375 on activities. On the last day of their vacation they decide to go horseback riding. They have already spent \$130 and there are five people in the family.

a. Horseback riding without a guide costs \$20 an hour per person. Write an inequality for the number of hours the Beard family could go horseback riding without a guide.....

b. Horseback riding with a guide costs \$26 an hour per person. Write an inequality for the number of hours the Beard family could go horseback riding with a guide.

Lesson
2.1.1

Rational Numbers

California Standards: Number Sense 1.3, 1.4, 1.5

1. Write each number in the form $\frac{a}{b}$.

The number 1.4 can be read aloud as "1 and 4 tenths."
The "tenths" indicates what number to place in the denominator.



a. 7

b. -12

c. 1.4

d. 0.58

e. 1.75

2. Explain why $\sqrt{2}$ is not a rational number.

3. Convert each fraction to a decimal.

a. $\frac{4}{5}$ -----

b. $\frac{9}{2}$ -----

c. $\frac{7}{8}$ -----

4. Convert each fraction to a decimal.

a. $\frac{2}{9}$ -----

b. $\frac{7}{3}$ -----

c. $\frac{5}{6}$ -----

5. For which values of x , where $1 \leq x \leq 15$, is the fraction $\frac{x}{15}$ a repeating fraction?

6. Write a fraction whose decimal expansion is:

a. terminating -----

b. repeating -----

7. The Purtle family spent $\frac{12}{5}$ hours at the pool on Sunday.
Express the amount of time the Purples spent at the pool on Sunday as a decimal.

Example

Convert $\frac{7}{22}$ into a decimal.

Solution

Divide 7 by 22.

$$\begin{array}{r}
 0.\overline{318} \leftarrow \text{repeating digits} \\
 22 \overline{) 7.0000} \\
 \underline{66} \\
 40 \leftarrow \text{repeated remainder} \\
 \underline{22} \\
 180 \\
 \underline{176} \\
 40
 \end{array}$$

So, $\frac{7}{22}$ as a decimal is $0.\overline{318}$.

8. Convert $\frac{7}{11}$ into a decimal.

9. Convert each fraction to a decimal.

a. $\frac{4}{27}$

b. $\frac{4}{7}$

10. Franklin cycled $\frac{11}{12}$ miles on Saturday.

Express the number of miles Franklin cycled on Saturday as a decimal.

Lesson
2.1.2

Converting Terminating Decimals to Fractions

California Standards: Number Sense 1.5

1. Write these numbers in words.

a. 5.8

b. 6.02

c. 9.45

Example

Convert 0.35 to a fraction in its simplest form.

Solution

0.35 is thirty-five hundredths. So, as a fraction it is $\frac{35}{100}$.

The greatest common factor (GCF) of 35 and 100 is 5.

$$\frac{35 \div 5}{100 \div 5} = \frac{7}{20}$$

2. Complete the table.

Fraction	Decimal	Terminating or Repeating
$\frac{52}{100}$		
$\frac{4}{11}$		
	0.5	
	0.904	
	4.3	

3. One student's response to a question on a test is $\frac{4}{6}$. Another student's response to the same question is $\frac{12}{18}$. How can their different answers be explained?
-

4. Convert each decimal to a fraction in its simplest form.

Read each number aloud to help identify the place value of the final digit.

a. 0.74

b. 0.04

c. 2.9

d. 0.077

5. Joey spent 3.25 hours on his science project during the weekend.

Express the amount of time Joey spent as a fraction in its simplest form. -----

6. Identify the GCF for each set of numbers:

Find the prime factorization for each number.

a. 24 and 36

b. 14 and 7

c. 22 and 33

d. 8 and 12

7. Write each of the following fractions in its simplest form.

a. $\frac{12}{48}$

b. $\frac{12}{30}$

c. $\frac{28}{32}$

d. $\frac{66}{200}$

8. Convert each decimal to a fraction in its simplest form.

a. 0.05

b. 2.2

c. 0.46

d. 3.4

9. The height of a cell phone tower is 95.75 feet. Convert this number to a fraction in simplest form.

10. Explain why decimals greater than 1 become improper fractions when converted.

11. Convert each decimal to a fraction in its simplest form.

a. 5.065

b. -0.52

c. 5.675

d. -1.284

Lesson
2.1.3

Converting Repeating Decimals to Fractions

California Standards: Number Sense 1.5

1. Explain what is meant by the idea of using subtraction to make repeating decimals “disappear.”

.....

.....

2. Find $14.\overline{72} - 2.\overline{72}$

3. Complete the table.

Fraction	Decimal	Terminating or Repeating
$\frac{10}{11}$		
$\frac{6}{5}$		
	0.595959...	

Example

Convert $0.555\ldots$ to a fraction.

Solution

$$\text{Let } x = 0.555\ldots$$

$$10x = 5.555\ldots$$

$$10x - x = 5.555\ldots - 0.555\ldots$$

$$9x = 5$$

$$\frac{9x}{9} = \frac{5}{9}$$

$$x = \frac{5}{9}$$

Remember, two equations are necessary to solve this problem.

4. If $x = 0.\overline{30}$, find: (i) $100x$ and (ii) $99x$. Use your results to write x as a fraction in its simplest form.

.....

5. Convert each repeating decimal to a fraction in simplest form.

a. $0.\overline{57}$

b. $0.\overline{743}$

c. $0.\overline{63}$

6. Are $0.\overline{10}$ and $0.\overline{1}$ equivalent decimals? Explain your answer.

7. When asked to convert $0.111\dots$ to a fraction, Michael's answer was $\frac{1}{8}$. Is Michael's answer correct? Explain your reasoning.

8. Convert each repeating decimal to a fraction in its simplest form.

a. $42.\overline{8}$

b. $4.\overline{530}$

c. $20.\overline{67}$

9. Marissa's math homework had a problem for which the answer was $21.\overline{25}$. She was asked to convert this decimal to a fraction in its simplest form. What should her answer have been?

10. Convert $21.\overline{275}$ to a fraction in its simplest form. -----

11. Convert each repeating decimal to a fraction in its simplest form.

a. $3.1\overline{08}$

b. $0.2\overline{65}$

c. $7.5\overline{724}$

Lesson 2.2.1

Absolute Value

California Standards: Number Sense 2.5

Example

Find the absolute value of each number.

a. $|-15|$

The absolute value of negative fifteen is the distance from 0 to negative fifteen on the number line.

b. $|3.5|$

Absolute value is the distance from zero to the number whether the number is an integer, a fraction, or a decimal.

Solution

a. 15

b. 3.5

1. Find the absolute value of each number.

a. $|2|$

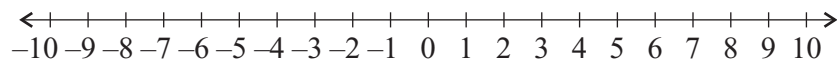
b. $|-5.5|$

c. $|-130|$

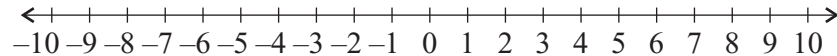
d. $|\frac{2}{5}|$

2. Show the solutions of the following equations on a number line.

a. $|x| = \frac{7}{5}$



b. $|x| = 6$



3. Explain why $|x| = -7$ does not have a solution.

4. Solve each equation.

a. $x = |-3|$

b. $x = |0|$

c. $|r| = 6$

d. $|z| = 45$

5. True or False:

a. $m = -18$ is a solution of the equation $|m| = 71$.

True

☐

False

☐

b. $m = -71$ is the only solution to the equation $|m| = 71$.

☐
☐

c. $k = -23$ is a solution to the equation $23 = |k|$.

☐
☐

b. $t = -17$ and $t = 11$ are both solutions to the equation $|t + 3| = 14$.

☐
☐

Find the absolute value of the difference of the two numbers.

6. Find the distance between each pair of numbers.

a. 4 and -9

b. -22 and -56

c. 0 and 16

d. -14 and 29

7. Emmanuel, a running back, started a football game struggling against the opposing team. He lost 5 yards on the first play and lost 8 more yards on the second play. What is the total amount of yards lost on the first two plays of the football game?

8. Simplify each expression.

a. $|14 - 6|$

b. $|6 - 14|$

c. $|-7 - 8|$

d. $|-9 + 3| + |-10|$

e. $-|-12| + |8 - 7|$

f. $14 - 3|4 - 6|$

g. $-|3 - 4| + 5|8 - 12|$

9. The distance between two points is 25. Find the other two possible endpoints when one endpoint is -32 .

10. If $r = 6$, $t = -2$, and $u = -1$, then evaluate each expression.

a. $|rt - u|$

b. $|u - t - r|$

c. $-|t^2 - ur|$

d. $|t + r| - |u - r|$

11. Evaluate each expression.

a. $|-45|$ -----

b. $|3 + 13 - 7|$ -----

12. Solve for x .

a. $x = |-24|$ -----

b. $|x| = 19$ -----

Lesson
2.2.2

Using Absolute Value

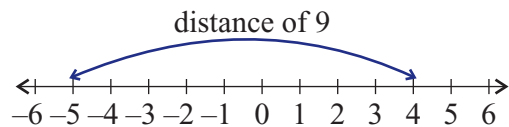
California Standards: Number Sense 2.5, Algebra and Functions 1.1

Example

Find the distance between -5 and 4 .

Solution

The distance between -5 and 4 is $|-5 - 4| = |-9| = 9$



1. Find the distance between each pair of numbers.

a. 25 and 18 b. -1.4 and 2.6 c. 7 and -3

2. A diver stands on a high diving board 12 m above the surface of the water in a pool. The diver dives into the pool vertically and reaches a depth of 4 m below the surface. How far did the diver travel?

.....

3. The table shows the daily high temperatures in St. Louis during one week in March.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
55 °F	62 °F	58 °F	61 °F	53 °F	58 °F	60 °F

- a. Between which two days did the high temperature change the least?

.....

- b. By how many degrees did the temperature vary during this week?

4. The mayor of town A estimates the town population between each census. In 1990, town A had a population of 3327. The table below shows the estimated increase or decrease per year until the next census in 2000.

1991	1992	1993	1994	1995	1996	1997	1998	1999
-125	-2	+83	-6	+1	+43	+9	+14	-11

- a. Between which two years did the estimated population decrease the most in town A?

.....

- b. Between which two years did the estimated population change least in town A?

.....

- c. What is the estimated population in 1999?

5. The ideal temperature, t , of a baby's bathwater should be no more than 5°F either way from 95°F . The expression $|95 - t|$ can be used to determine if the bathwater is at a good temperature.

Emma runs a bath with a water temperature of 92°F . Is this a good temperature to bath her baby?

6. A candy manufacturer aims for each of its bags of candy bars to contain 33 bars. Each bag can vary, having up to three more or three less candy bars.
- a. Write an inequality to represent the acceptable range of candy bars per bag, using x to represent the number of bars in a bag.
-

- b. Graph the acceptable range of candy bars per bag.



7. The length of a surfboard for a certain competition category should be no more than 0.75 m away from 3 meters . The expression $|3 - l|$ is used to check whether a surfboard of length l meters fit the size requirement.

Determine whether a surfboard measuring 2.55 meters is within the correct range.

8. On average a knitter will use 975 yards of yarn to knit a child's pullover. Depending on the size of the pullover, the amount of yarn used will vary by about 200 yards. The expression $|975 - y|$ can be used to check whether a knitter has enough yarn to knit a child's pullover.

Su Lin has 725 yards of yarn. Does she have enough to knit a child's pullover?

9. The amount of free available chlorine, c , in a swimming pool should be less than 1.0 ppm away from 2.0 ppm . Quambo uses the expression $|2.0 - c|$ to check that the level of free available chlorine is within these limits.

Is a swimming pool with a free available chlorine level of 4.6 ppm within the limits?

Lesson
2.3.1

Adding and Subtracting Integers and Decimals

California Standards: Number Sense 1.2

Example

Simplify: a. $4 + 3$

Count the spaces as you move right or left along the number line.

b. $-4 + 5$

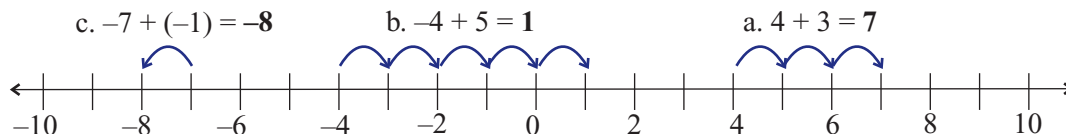
Begin at -4 then move 5 spaces to the right on the number line.

c. $-7 + (-1)$

Begin at -7 then move 1 space to the left on the number line.

Remember when adding positive numbers to move to the right and when adding negative numbers to move to the left on the number line.

Solution



1. Calculate the following sums.

a. $5 + (-8)$

b. $6 + (-2)$

c. $(-18) + (-1)$

d. $-5 + (-5)$

e. $-15 + 7$

f. $4 + (-10)$

g. $-13 + 0$

h. $-7 + 4 + (-3)$

i. $12 + 10 + (-17)$

j. $3 + (-8) + 12$

2. a. Use the number line to explain why $-5 + 5 = 0$.



- b. Explain why $-x + x = 0$ for every x except when x equals 0.

3. Identify the additive inverse of each number.

a. 7

b. -15

c. -90

4. Calculate the following sums.

a. $5 - 1$

b. $-7 - 5$

c. $17 - (-4)$

d. $-11 - (-8)$

e. $-3 - 2 - (-10)$

f. $12 - (-4) - 8$

Subtracting a negative number is the same as adding a positive one.

5. Evaluate each expression when $x = -7$, $y = 4$, and $z = -2$.

- a. $x + y$

- b. $y - z$

- c. $y - x + z$

- d. $x - x$

6. An internet pet supply store sells cat scratching posts ranging in height from 20 inches to 36 inches. What is the difference in height between the tallest and shortest cat scratching posts?

7. Calculate.

- a. $65 + 82$

- b. $53 + 26$

- c. $672 + 437$

- d. $478 + 463$

8. Calculate.

- a. $5.89 + 8$

- b. $2.09 + 4.8$

- c. $10 + 2.76$

- d. $3.89 + 9.4$

9. Calculate.

- a. $8.25 - (-3.76)$

- b. $4.87 - (-7.4)$

- c. $18 - 12.2$

- d. $27 - 14.64$

- e. $10.6 - 5.349$

- f. $8.12 - (-23) - 2.3$

- g. $9.3 - 7.2 - 1.9$

10. The table shows the precipitation in inches during the previous 24 hours, 4 days, 21 days, and 28 days on a river.

24 hours	4 days	21 days	28 days
1.3	1.67	3.34	4.55

- a. How much precipitation fell between 5 and 28 days ago? -----
- b. The next day, 0.87 inches of precipitation was recorded.
What was the total precipitation in inches for all 29 days? -----

11. Calculate.

- a. $6 + (-17)$ -----
- b. $-5.6 + (-4.02)$ -----

12. Calculate.

- a. $-8.25 - 5$ -----
- b. $-72.3 - 21.4$ -----

13. A pet supply company sells steps for dogs to aid them in entering cars or trucks. The height of a single step is 8.75 inches, double steps are 13.25 inches, and triple steps are 20 inches.

a. What is the difference in height between the double and single steps? -----

b. What is the difference in height between the triple step and single step? -----

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Lesson
2.3.2

Multiplying and Dividing Integers

California Standards: Number Sense 1.2

1. Calculate the following multiplications.

a. -7×4

b. $-1 \times (-11)$

c. 9×5

d. $3 \times (-3)$

e. $5 \times (-6) \times 2$

f. $-3 \times (-10) \times (-2)$

g. $-9 \times 0 \times 4$

The product of a negative number and a positive number is a negative number.

2. Calculate the following divisions.

a. $-14 \div 2$

b. $-90 \div -3$

c. $64 \div 8$

d. $-36 \div -4$

e. $55 \div -5$

f. $72 \div -9 \div -2$

g. $-45 \div -5 \div -3$

The same rules for multiplication of integers apply for division of integers.

3. a. Complete the table:

$(-3)(-3)$	=
$(-3)(-3)(-3)$	=
$(-3)(-3)(-3)(-3)$	=
$(-3)(-3)(-3)(-3)(-3)$	=
$(-3)(-3)(-3)(-3)(-3)(-3)$	=

- b. Will the product of an even number of negative numbers be a negative or positive number?

4. a. An annual crossword puzzle tournament has 500 participants. The entry fee is \$145 for each participant. What is the total amount of money collected for all 500 participants?

- b. One crossword puzzle enthusiast is able to complete 6 puzzles in 42 minutes. If she averages the same amount of time on each crossword puzzle then how long does it take this crossword enthusiast to solve one crossword puzzle?

Multiply the numbers then determine whether the product is negative or positive.

5. Calculate the following multiplications.

a. 45×-7

b. -302×-28

c. 65×91

d. -345×73

6. Calculate the following divisions.

a. $-845 \div 5$

b. $-744 \div -31$

c. $192 \div 12$

d. $85 \div -5$

7. Sawyer bought 15 plants for \$270. How much was each plant on average?

8. Mrs. Stukel and her two daughters are going to the movies. Each ticket costs \$8.75. How much will it cost Mrs. Stukel and her two daughters to attend the movies?

9. Calculate the following multiplications.

a. $-2 \times -5 \times -4$

b. 34×203

10. Calculate the following divisions.

a. $2744 \div -49$

b. $11,684 \div -46$

11. Calculate.

a. $14 \times -8 \times -86$

b. $-864 \div (24 \div 8)$

c. $(90 \div 6) \times -15$

d. $(-31 \times -68) \div 2$

e. $12 \times -13 \times -224$

f. $(75 \div -5) \times 63$

Lesson
2.3.3

Multiplying Fractions

California Standards: Number Sense 1.2

1. Calculate these fractions by drawing area models.

a. $\frac{1}{4} \times \frac{2}{5}$

b. $\frac{1}{5} \times \frac{3}{4}$

2. Calculate the following, giving your solutions in their simplest form.

It is not necessary to find a common denominator when multiplying fractions.

a. $-\frac{7}{10} \times \frac{2}{6}$

b. $-\frac{2}{5} \times -\frac{4}{13}$

c. $\frac{6}{7} \times \frac{1}{5}$

d. $-8 \times \frac{3}{16}$

e. $-\frac{7}{2} \times -\frac{4}{9}$

Example

Calculate $3\frac{2}{9} \times -2\frac{1}{6}$.

Solution

$$3\frac{2}{9} = \frac{(3 \times 9) + 2}{9} = \frac{29}{9} \text{ and } -2\frac{1}{6} = -\frac{(2 \times 6) + 1}{6} = -\frac{13}{6}, \text{ so}$$

$$\begin{aligned} 3\frac{2}{9} \times -2\frac{1}{6} &= \frac{29}{9} \times -\frac{13}{6} \\ &= -\frac{29 \times 13}{9 \times 6} \\ &= -\frac{377}{54} \end{aligned}$$

3. Calculate the following. Give your solutions in their simplest form.

a. $4\frac{2}{5} \times 2\frac{1}{6}$

b. $-7\frac{2}{9} \times 1\frac{4}{5}$

c. $-3\frac{4}{7} \times -4\frac{1}{2}$

d. $3\frac{1}{4} \times -2\frac{3}{11}$

4. When asked how the product of a whole number and a fraction smaller than 1 compared to the original whole number, Elise responded, "The product is always larger than the original number." Is Elise correct? Explain your response.

5. If each book has a weight of $1\frac{1}{4}$ pounds, determine the weight of a stack of eight books.

6. Calculate the following. Give your solutions in their simplest form.

a. $\frac{2}{5} \times -\frac{6}{7}$

b. $1\frac{2}{3} \times -5\frac{1}{2}$

c. $3\frac{3}{4} \times -5\frac{1}{5}$

d. $-2\frac{4}{5} \times -\frac{6}{7}$

e. $-\frac{6}{10} \times -\frac{5}{7}$

f. $9\frac{1}{3} \times \frac{8}{9}$

7. There was $\frac{5}{8}$ of a gallon of paint in a bucket. Manny used $\frac{3}{4}$ of that. How much paint was left?

8. To calculate the product $\frac{2}{3} \times \frac{4}{9}$, Ivan put both fractions over a common denominator, giving $\frac{6}{9} \times \frac{4}{9}$, then multiplied numerators and denominators separately. Would this produce an incorrect answer?

9. Sade had $\frac{3}{5}$ of her book left to read, then on Tuesday night she read $\frac{3}{4}$ of the unread portion.

What portion of the whole book did she read on Tuesday night?

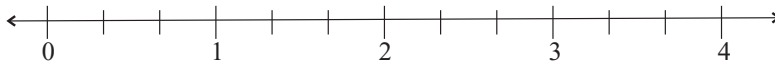
10. In a bicycle race, $\frac{1}{3}$ of the course is road. If the course is $1\frac{1}{4}$ miles long, determine how much of the course is road.

Lesson 2.3.4

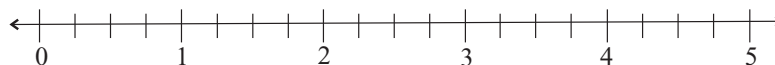
Dividing Fractions

California Standards: Number Sense 1.2

1. Show $4 \div \frac{1}{3}$ on the number line.



2. Show $5 \div \frac{1}{2}$ on the number line.



3. Complete the table.

Fraction	Reciprocal
$-\frac{4}{5}$	
	$-\frac{11}{2}$
$-\frac{5}{3}$	
-9	
	$-\frac{4}{7}$

4. Convert each division problem into a multiplication problem.

a. $\frac{1}{7} \div \frac{1}{5}$

b. $\frac{3}{4} \div \frac{1}{6}$

c. $\frac{8}{6} \div \frac{2}{3}$

d. $\frac{5}{8} \div \frac{3}{5}$

Example

Calculate $-2\frac{3}{4} \div 1\frac{2}{5}$.

Solution

$$-2\frac{3}{4} = -\frac{(2 \times 4) + 3}{4} = -\frac{11}{4} \text{ and } 1\frac{2}{5} = \frac{(1 \times 5) + 2}{5} = \frac{7}{5}$$

$$-2\frac{3}{4} \div 1\frac{2}{5} = -\frac{11}{4} \div \frac{7}{5} = -\frac{11}{4} \times \frac{5}{7}$$

$$-\frac{11}{4} \times \frac{5}{7} = -\frac{11 \times 5}{4 \times 7} = -\frac{55}{28}$$

Convert the mixed fractions to improper fractions.

Write the division as a multiplication.

Calculate the multiplication.

5. Simplify.

a. $-\frac{1}{5} \div \frac{2}{3}$

b. $-6 \div -\frac{4}{9}$

c. $\frac{2}{13} \div \frac{4}{5}$

d. $\frac{12}{13} \div 6$

e. $-\frac{3}{7} \div -\frac{9}{11}$

6. Calculate the following divisions. Where possible, write each solution as a mixed number.

a. $6 \div 2\frac{1}{3}$

b. $\frac{10}{11} \div 7\frac{1}{5}$

c. $2\frac{1}{3} \div 3\frac{1}{2}$

d. $1\frac{4}{9} \div 6\frac{7}{8}$

7. Elsa walks $4\frac{1}{4}$ miles in $1\frac{1}{2}$ hours. How fast does she walk in miles per hour? -----

8. An apple grower will ship $\frac{3}{4}$ tons of apples in two equal shipments.

How much will each shipment weigh? -----

9. A carpenter is building a shelf that is $3\frac{1}{8}$ ft long.

How many $\frac{3}{4}$ ft boxes can he store on the shelf? -----

10. A state highway department is installing distance markers every $\frac{2}{10}$ of a mile at a cost of \$255.50 per sign. How much will it cost the state highway department to install 10 miles of these markers?

11. Simplify.

a. $-4 \div 2\frac{3}{5}$

b. $-3\frac{1}{6} \div -2\frac{4}{7}$

c. $4\frac{1}{3} \div -1\frac{2}{3}$

d. $7\frac{1}{4} \div 8$

12. Simplify.

a. $-\frac{4}{11} \div \frac{8}{33}$ -----

b. $5\frac{1}{3} \div 2\frac{2}{5}$ -----

13. A rectangular table has an area of $16\frac{1}{24}$ ft². The length of the table is $4\frac{3}{8}$ ft.

What is the width of the table? -----

Lesson
2.3.5

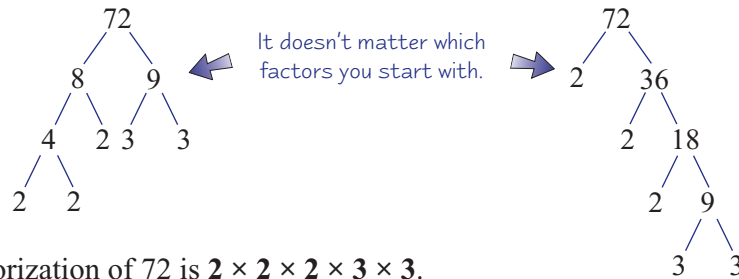
Common Denominators

California Standards: Number Sense 1.1, 1.2, 2.2

Example

Find the prime factorization of 72.

Solution



The prime factorization of 72 is $2 \times 2 \times 2 \times 3 \times 3$.

1. Find the prime factorization of each number.

a. 9

b. 12

c. 17

d. 15

.....

.....

.....

.....

2. Find the prime factorization of each number.

a. 81

b. 45

c. 39

.....

.....

.....

d. 28

e. 64

f. 135

.....

.....

.....

3. a. What are the first five multiples of 6?

.....

b. What are the first five multiples of 10?

.....

c. What is the least common multiple of 6 and 10?

.....

4. Find the least common multiple (LCM) for each set of numbers.

a. 35 and 21

b. 60 and 36

c. 54 and 36

d. 24, 30, and 42

5. a. Find two numbers with an LCM of 54.

b. Find three numbers with an LCM of 48.

c. Find two numbers with an LCM of 70.

d. Find three numbers with an LCM of 84.

6. Find the least common denominator for each pair of fractions.

a. $\frac{2}{5}$ and $\frac{3}{10}$

b. $\frac{3}{8}$ and $\frac{5}{12}$

c. $\frac{4}{9}$ and $\frac{11}{27}$

d. $\frac{4}{7}$ and $\frac{8}{9}$

7. Find the greater fraction in each pair by finding a common denominator.

a. $\frac{1}{5}$ and $\frac{2}{9}$

b. $\frac{7}{8}$ and $\frac{2}{3}$

c. $\frac{11}{13}$ and $\frac{6}{7}$

8. Complete the expressions using $<$, $>$, or $=$.

a. $\frac{4}{5}$ $\frac{1}{9}$

b. $\frac{11}{14}$ $\frac{14}{15}$

c. $\frac{16}{17}$ $\frac{18}{19}$

9. Order these fractions from least to greatest: $\frac{1}{16}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{8}$.

Lesson
2.3.6

Adding and Subtracting Fractions

California Standards: Number Sense 1.2, 2.2

1. Rewrite the fractions with a common denominator.

a. $\frac{4}{5}$ and $\frac{7}{15}$

b. $\frac{5}{12}$ and $\frac{5}{18}$

c. $\frac{11}{36}$ and $\frac{7}{24}$

Example

Calculate $\frac{6}{7} + \frac{2}{3}$.

Solution

First make two fractions that have the same denominator.

$$\frac{6}{7} \times \frac{3}{3} = \frac{18}{21} \text{ and } \frac{2}{3} \times \frac{7}{7} = \frac{14}{21}$$

Then add the two fractions.

$$\frac{18}{21} + \frac{14}{21} = \frac{32}{21}$$

2. Calculate $\frac{2}{3} + \frac{4}{5}$

3. Calculate $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$

4. Jose mowed $\frac{1}{7}$ of the lawn. Later, Norma mowed $\frac{1}{3}$ of the lawn.

How much of the lawn did they mow together?

5. Clinton baked a dozen cupcakes. He put chocolate frosting on $\frac{1}{2}$ of the cupcakes and vanilla frosting on a further $\frac{1}{4}$ of the cupcakes. He left the rest plain. What fraction of Clinton's cupcakes were frosted?

.....

6. Luis rides his bike home from work each day. He lives $\frac{3}{4}$ of a mile from work.
 Today after riding $\frac{1}{2}$ mile he makes a stop at a coffee shop.
 Determine how far Luis is from his house when he is at the coffee shop.

7. Stuart has $\frac{3}{4}$ of a tank of gas in his automobile. After driving to and from work his tank is $\frac{7}{12}$ full.
 What fraction of the tank of gas does Stuart use to drive to and from work?

8. Calculate the following, and express all answers in their simplest form.

a. $\frac{6}{7} - \left(-\frac{2}{7}\right)$

.....

b. $-\frac{8}{19} - \left(-\frac{9}{19}\right)$

.....

c. $\frac{8}{15} - \frac{13}{15}$

.....

9. Explain the error in the following calculation: $\frac{5}{11} + \frac{5}{17} = \frac{5}{28}$.

.....

10. Calculate the following, and express all answers in their simplest form.

a. $\frac{5}{7} + \left(-\frac{3}{4}\right)$

.....

b. $-\frac{2}{9} + \left(-\frac{3}{5}\right)$

.....

c. $\frac{1}{6} + \frac{11}{12}$

.....

d. $-\frac{5}{6} + \left(-\frac{1}{2}\right)$

.....

e. $\frac{4}{9} + \left(-\frac{7}{18}\right) + \frac{5}{36}$

.....

f. $-\frac{3}{8} + \left(-\frac{1}{6}\right) + \left(-\frac{5}{12}\right)$

.....

11. Calculate the following, and express all answers in their simplest form.

a. $\frac{5}{12} - \frac{2}{9}$

.....

b. $-\frac{11}{15} - \frac{17}{30}$

.....

c. $\frac{6}{7} - \left(-\frac{4}{5}\right)$

.....

d. $-\frac{1}{2} - \left(-\frac{3}{5}\right)$

.....

e. $-\frac{4}{9} - \frac{2}{3} - \left(-\frac{5}{8}\right)$

.....

f. $\frac{1}{5} - \left(-\frac{5}{6}\right) - \left(-\frac{3}{7}\right)$

.....

Lesson

2.3.7 Adding and Subtracting Mixed Numbers

California Standards: Number Sense 1.2, 2.2

1. Write the mixed numbers as improper fractions:

a. $4\frac{3}{8}$

b. $7\frac{2}{7}$

c. $2\frac{11}{12}$

d. $7\frac{1}{5}$

ExampleFind $3\frac{1}{8} - 2\frac{4}{5}$.**Solution**

$$3\frac{1}{8} = \frac{(3 \times 8) + 1}{8} = \frac{25}{8} \text{ and } 2\frac{4}{5} = \frac{(2 \times 5) + 4}{5} = \frac{14}{5}$$

$$\frac{25}{8} \times \frac{5}{5} = \frac{125}{40} \text{ and } \frac{14}{5} \times \frac{8}{8} = \frac{112}{40}$$

$$\frac{125}{40} - \frac{112}{40} = \frac{13}{40}$$

Use $8 \times 5 = 40$.

Convert the mixed numbers to fractions

Find a common denominator

Subtract the fractions

2. Find:

a. $2\frac{2}{3} + 5\frac{3}{4}$

b. $3\frac{1}{3} + 6\frac{3}{8}$

3. Find $\frac{6}{5} - \frac{1}{4} + 3\frac{1}{2}$

4. Complete the following calculations. Express your solutions in their simplest form.

a. $1\frac{3}{5} + 4\frac{1}{5}$

b. $2\frac{4}{12} - 1\frac{3}{12}$

c. $1\frac{3}{4} + 2\frac{5}{6}$

d. $3\frac{2}{3} - 2\frac{7}{9}$

e. $1\frac{5}{7} + 1\frac{2}{5}$

f. $2\frac{1}{4} - 4\frac{5}{24}$

5. Find $5\frac{1}{6} + 2\frac{1}{6}$. Give your answer in its simplest form.

6. Jodi is going to Rosa's house. She walks the first $1\frac{1}{2}$ miles. Then she jogs the remaining $1\frac{1}{4}$ miles.

How far does Jodi live from Rosa?

7. Rashad plans to add a chair rail to his kitchen wall at a height of $33\frac{1}{4}$ inches from the floor. After measuring the height of the kitchen chair backs, Rashad needs to raise the chair rail to $34\frac{1}{8}$ inches.

By how many inches will Rashad have to adjust his measurements to fit the new chair rail height?

.....

8. A farmer has just harvested the honey from his beehives. The harvested honeycomb weighs $4\frac{1}{8}$ pounds. After separating the honey from the comb, he has $3\frac{1}{4}$ pounds of honey.

Determine the weight of the comb.

9. Simplify the following and express your answers in their simplest form.

a. $-8\frac{3}{8} - \left(-6\frac{5}{6}\right)$

b. $1\frac{4}{11} - \left(-3\frac{3}{4}\right)$

.....

.....

10. In the fall, Victoria measured 3 feet $5\frac{1}{4}$ inches tall. Ten months later she measured 3 feet $7\frac{3}{8}$ inches.

By how many inches did Victoria grow in ten months?

Lesson

2.4.1

Further Operations with Fractions

California Standards: Number Sense 1.2, 2.2

Example

Calculate $\frac{3}{7} + \frac{4}{3} \times \frac{9}{2}$.

Solution

$$\frac{3}{7} + \frac{4 \times 9}{3 \times 2} = \frac{3}{7} + \frac{36}{6}$$

Since there are no parentheses, do the multiplication before the addition

The LCM of 6 and 7 is 42, so use this as the common denominator.

$$\frac{3}{7} = \frac{3 \times 6}{7 \times 6} = \frac{18}{42} \text{ and } \frac{36}{6} = \frac{36 \times 7}{6 \times 7} = \frac{252}{42}, \text{ so } \frac{3}{7} + \frac{36}{6} = \frac{18}{42} + \frac{252}{42} = \frac{18 + 252}{42} = \frac{270}{42} = \frac{45}{7}.$$

1. Calculate each of the following.

a. $\frac{3}{4} + \frac{6}{2} \times \frac{8}{3}$

b. $\frac{20}{36} \div \frac{1}{3} - 2\frac{1}{6}$

2. Calculate each of the following.

a. $\left(3\frac{1}{3} + \frac{1}{4}\right) \times \frac{1}{2}$

b. $\left(2\frac{1}{5} + \frac{4}{5}\right) - 2 \times \frac{1}{6}$

3. Evaluate the expression $\frac{1}{3} \div 2\frac{1}{4} \times \left(2 + 3\frac{1}{6}\right)$

4. Ada has $3\frac{1}{2}$ cups of pumpkin seeds. Her friend adds $\frac{3}{4}$ of a cup of seeds. Ada wants to put the seeds in bags that can hold $\frac{1}{2}$ a cup each. How many bags can she fill?

5. A theater group had $5\frac{3}{4}$ yards of fabric for new costumes. Someone donated 4 more yards.

How many costumes can be made if each costume takes $\frac{3}{8}$ of a yard?

6. Joanna has 3 bags of beans that are each half full. When full, each bag weighs $\frac{3}{4}$ of a pound.

Determine the total weight of her three bags.

7. When asked to simplify the expression $3\frac{1}{4} \times \frac{1}{8} \div 2$, Alison's answer was $\frac{8}{32}$.

Is Alison's answer correct? Show your working.

.....

.....

.....

8. Tamisha has $2\frac{1}{2}$ pounds of peanuts that she wants to divide equally into four bags. Determine the amount of peanuts each of the four bags will have.

9. Calculate $\frac{\frac{1}{3}}{\frac{3}{3\frac{3}{4}}} \times \left(2\frac{1}{6} - \frac{1}{2}\right)$

10. Calculate the following. Give your answers in their simplest forms.

a. $\frac{\frac{2}{5} + \frac{1}{2}}{\frac{3}{10}} - \left(1\frac{4}{5} \times \frac{1}{3}\right)$

.....

b. $\frac{\frac{3}{7}}{\frac{1}{3} - 2\frac{5}{6}} \times \frac{2}{3} + \frac{6}{7}$

.....

11. Ruth makes $6\frac{1}{2}$ pounds of jam, and gives a third of it away. She then makes an additional $4\frac{1}{4}$ pounds of jam. She mixes the two batches and divides the jam into jars, each containing $\frac{3}{4}$ pounds.

How many jars of jam does she have?

Lesson

2.4.2 Multiplying and Dividing Decimals

California Standards: Number Sense 1.2

1. Calculate the following using area models.

a. 0.3×0.4

b. 0.2×0.9

ExampleCalculate 1.6×0.05 .**Solution**

$$1.6 \times 0.05 = \frac{16}{10} \times \frac{5}{100} \quad \leftarrow \text{Write both decimals as fractions.}$$

$$= \frac{16 \times 5}{10 \times 100} \quad \leftarrow \text{Multiply them.}$$

$$= \frac{80}{1000} = 80 \div 1000 = 0.08 \quad \leftarrow \text{Divide by 1000 to get the decimal answer.}$$

2. Rewrite the following decimal multiplications as fraction calculations.

a. 2×1.3

b. 3.4×-6.7

c. 1.02×3.26

d. -4.6×9.45

3. Solve the following by writing them as fraction calculations:

a. 4×6.3

b. 3×4.7

c. -2×2.5

d. -6.34×-9

4. Solve the following by writing them as fraction calculations:

a. 7.5×4.6

b. -9.6×-7.4

c. -6.22×1.25

d. 2.24×8.61

5. Solve the following by writing them as fraction calculations:

a. 5.4×9.42

b. 5.13×-0.6

c. -6.42×-7.2

d. 2.1×2.345

6. Zulfi calculated the product of 2.1468×4.264 as 91.539552. Is he correct? Explain.

7. Solve the following by writing them as fraction calculations:

a. $6 \div 1.5$

b. $5.67 \div 3$

c. $1.43 \div 1.3$

d. $6.45 \div 0.03$

8. Solve the following divisions.

a. If $4 \div 8 = 0.5$, calculate $0.04 \div 8$

b. If $6 \div 32 = 0.1875$, calculate $0.6 \div 0.32$

c. If $55 \div 5 = 11$, calculate $0.0055 \div 0.00005$

9. Solve these divisions using integer division.

a. $4.06 \div 1.4$

b. $1.054 \div 3.4$

c. $1.696 \div 0.53$

d. $0.6893 \div 11.3$

10. A supermarket sells bananas at \$2.64 per pound. How much would 0.5 pounds of bananas cost?

11. Gabrielle fills up her car with 30.4 liters of gasoline, which costs her 31.92 dollars.

How much is the gasoline per liter? -----

Lesson
2.4.3

Operations with Fractions and Decimals

California Standards: Number Sense 1.2

Example

Calculate $0.75 \times \frac{4}{5}$ by:

- a. converting 0.75 to a fraction, b. converting $\frac{4}{5}$ to a decimal.

Solution

a. 0.75 is $\frac{75}{100} = \frac{3}{4}$, so $0.75 \times \frac{4}{5} = \frac{3}{4} \times \frac{4}{5} = \frac{12}{20} = \frac{3}{5}$

b. $\frac{4}{5}$ is $4 \div 5 = 0.8$, so $0.75 \times \frac{4}{5} = 0.75 \times 0.8 = 0.6$

1. Calculate each of the following by: (i) converting the decimal to a fraction and (ii) converting the fraction to a decimal.

a. $0.35 \times \frac{2}{5}$ _____

b. $\frac{1}{4} \times 1.3$ _____

2. Calculate each using the easiest method.

a. $\frac{2}{3} - 0.8$ _____

b. $0.5 \times \frac{1}{8}$ _____

3. Calculate:

a. $\frac{2}{5} \times 2.5$ _____

b. $6.3 \times \frac{7}{9}$ _____

4. Calculate:

a. $\frac{1}{6}(4.25 - 2.05)$ _____

b. $2.6 \times \left(\frac{3}{5} - \frac{1}{5}\right)$ _____

5. Calculate:

a. $\frac{(3.25 + 11.4)}{\frac{3}{4}}$ _____

b. $\frac{(15.1 - 2.33)}{\frac{1}{5}}$ _____

6. Simplify the expressions.

a. $(-4.26 - 1.38) \times \frac{3}{5}$

b. $-2.3 - \frac{2}{5}$

7. When asked to simplify the expression $\frac{1}{4}(2.36 - 1.08)$, Amelia did the following: $1.28 \div 4 = 0.32$.

Sam said her answer must be incorrect because she divided instead of multiplying.

Is Amelia's answer correct or incorrect? Explain.

.....
.....

8. Calculate $0.60 \times \frac{2}{5}$ by:

a. converting 0.60 to a fraction.

b. converting $\frac{2}{5}$ to a decimal.

9. When asked to simplify the expression $4.25 \times \frac{6}{7}$, two students used different methods.

Student one first multiplied by 6, then divided by 7.

Student two first divided by 7, then multiplied by 6.

Will the students get different answers? If not then which student is correct? Explain.

.....
.....

10. David has a piece of string that is 6.24 meters long. He cuts it into quarters.

How long is each quarter?

11. Sarah buys 0.43 kilograms of apples and 0.77 kilograms of pears. She uses two-thirds of the fruit to make an apple and pear crumble. Find the mass of fruit that's used to make the crumble.

.....

Lesson
2.4.4

Problems Involving Fractions and Decimals

California Standards: Number Sense 1.2, Mathematical Reasoning 2.2

Example

The length of a rectangular game table is $10\frac{1}{2}$ feet, while its width is 4.2 feet.

What is the area of the game table?

Solution

Area = length \times width

$$= 10\frac{1}{2} \times 4.2$$

$$= \frac{21}{2} \times 4.2$$

$$= (4.2 \div 2) \times 21$$

$$= 2.1 \times 21$$

$$= 44.1$$

The game table has an area of **44.1 ft²**.

Remember to include the correct units in your answer.

1. The width of a rectangular box is 10.3 inches while its length is $23\frac{1}{2}$ inches.

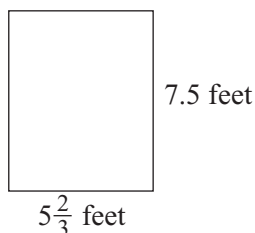
Find the area of the box.

2. A rectangular driveway is $4\frac{1}{3}$ yards wide and 6.5 yards long. Determine the area of the driveway.

.....

3. Find the area of the rectangle in the diagram.

.....



4. A piece of fabric 12.75 feet long needs to be divided into shorter lengths of $1\frac{1}{2}$ feet long.

How many full lengths can be made from this piece of fabric?

5. A batch of fertilizer requires $6\frac{1}{2}$ cups of powder to be mixed with water. A farmer wants a large area to be fertilized, so he uses $21\frac{3}{4}$ cups of powder. How many batches is this enough powder for?
-

6. In a recent election, $\frac{2}{3}$ of the citizens in a town voted. Of those voting, 60% voted in favor of a school bond. What fraction of the citizens voted for the school bond?

7. Romero trained 15.5 hours in one particular week. He spent $\frac{3}{4}$ of these hours running, while the rest of the time he spent weight training. How many hours did Romero spend weight training that week?
-

8. A farmer harvested $2\frac{1}{2}$ tons of pumpkins in a particular year. $\frac{2}{3}$ of them (by weight) were sent to a cannery, while the rest were sent to markets. Find how many tons of pumpkins were sent to markets.
-

9. Approximately $\frac{1}{3}$ of the dogs in a particular obedience training school are on the advanced level. Of those dogs, 40% are purebred. There are no purebred dogs in any of the other classes. Determine the fraction of the dogs in the school that are purebred.
-

10. Matthew won \$1502.72 in a lottery. He decided to spend $\frac{3}{4}$ of it on a holiday. How much money will he have left over?

11. A rectangular room has the dimensions $12\frac{1}{2}$ feet and 15.2 feet. $\frac{5}{8}$ of the room is carpeted. Find the area of the floor that is carpeted.

Lesson
2.5.1

Powers of Integers

California Standards: Number Sense 1.2, Algebra and Functions 2.1

1. Complete the table.

	Base	Exponent
6^8		
(-4)		
	7	5
	-6	2

Parentheses are important when the base is a negative number.

2. Write each of the following expressions as a power in base and exponent form.

a. 3×3

b. $7 \times 7 \times 7 \times 7$

c. 8

.....

.....

.....

3. Write each of the following expressions as a power in base and exponent form.

a. $-2 \times -2 \times -2$

b. -12×-12

c. $-1 \times -1 \times -1 \times -1 \times -1$

.....

.....

.....

4. Write each of the following in base and exponent form.

a. five to the fourth power

b. twelve to the power 2

c. minus 3 to the power 4

.....

.....

.....

Example

Evaluate 5^3 .

Solution

$5 \cdot 5 \cdot 5 = 125$

↖ The exponent, 3, indicates how many times to multiply the base, 5, by itself.

5. a. Identify the base and exponent of 8^3

- b. Evaluate 8^3

6. Evaluate the following expressions.

a. 6^2

b. $(-5)^3$

c. 7^3

d. 1^{20}

7. Jasmine and nine of her friends are giving away free samples of hot chocolate.

Each person gives away ten free samples. How many samples are given out in total? -----

8. Racquel wrote in her math homework that $(-2)^2 = -2^2$. Is she correct? Explain your answer.

9. A square has a side length of 4 cm.

a. Write the area of the square in base and exponent form. -----

b. Calculate the area of the square. -----

10. Gemma sends a chain email to three friends. Each person who receives the email sends it on to three of their friends. Gemma sends the email to Kyle, who sends it to Pilar, who sends it to Rosita.

a. Write an exponential expression with 3 as the base to describe how many times the email will have been sent by the time Pilar receives it. You can assume that each 'round' of sending emails happens at the same time.

b. How many times will the email have been sent by the time Rosita has received the email?

Lesson 2.5.2

Powers of Rational Numbers

California Standards: Number Sense 1.2, Algebra and Functions 2.1

Example

Evaluate $\left(\frac{1}{5}\right)^3$

Remember that you need to raise both the numerator and denominator separately to the same power.

Solution

$$\left(\frac{1}{5}\right)^3 = \frac{1^3}{5^3} = \frac{1}{125}$$

$1 \cdot 1 \cdot 1 = 1$
 $5 \cdot 5 \cdot 5 = 125$

1. Evaluate the following exponential expressions.

a. $\left(\frac{9}{10}\right)^2$

b. $\left(\frac{3}{8}\right)^3$

c. $\left(\frac{1}{4}\right)^5$

d. $\left(-\frac{6}{5}\right)^4$

2. Write the following expressions in base and exponent form.

a. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

b. $\frac{8}{9} \cdot \frac{8}{9} \cdot \frac{8}{9}$

c. $\frac{4}{3} \cdot \frac{4}{3}$

3. Complete the table.

	Base	Exponent	Answer
$(0.1)^3$			
$(0.2)^3$			
	0.5	2	
		2	0.64

4. Evaluate and simplify $(0.4)^3$. Show your working.

5. Evaluate the following expressions. Give your answer as a decimal.

a. $(0.02)^2$

b. $(0.41)^2$

c. $(0.16)^3$

d. $(0.25)^3$

6. Without calculation, how many decimal places will $(0.23)^9$ give? -----

7. Kerri cuts a pie into thirds, and then each slice into thirds.

How many slices is the pie cut into? -----

8. Rosemarie is sharing out \$8 equally between her grandchildren.

She splits the \$8 between her four sons. Each son then splits his share between his four children.

a. How many grandchildren does Rosemarie have? -----

b. How much money does each grandchild receive? -----

9. Lisa gets paid \$525 each month. She splits her wages into fifths, and then splits each fifth equally into five envelopes to help her budget her money.

a. What fraction of her monthly wage is each envelope? -----

b. How much money is in each envelope? -----

10. Mrs. Cayse offers Hayden two choices for his allowance over the summer vacation:

Option 1: \$1 in the first week, then each week's allowance being double the previous week's.

Option 2: \$1 in the first week, then each week's allowance being the square of the number of dollars he received the previous week.

Which option should Hayden choose? -----

Lesson
2.5.3

Uses of Powers

California Standards: Number Sense 1.1, 1.2

1. Decide whether each of the following statements is true or false.

True False

a. If the area of a square is 49 inches², then the side length of the square is 7 inches.

☐ ☐

b. If the side length of a square is 100 cm, then the area of the square is 1000 cm².

☐ ☐

c. If the side length of a square is $\frac{7}{8}$ ft, then the area of the square is $\frac{14}{8}$ ft².

☐ ☐

d. If the area of a square is 0.01 yd², then the side length of the square is 0.1 yd.

☐ ☐

2. Find the area of a square with the following side lengths.

Remember to include the appropriate units in your answer.

a. 6 cm

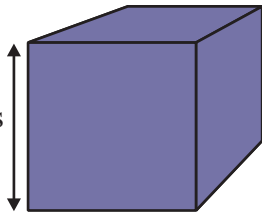
b. 4.5 in.

c. 6.2 ft

3. Find the volume of the following cubes.

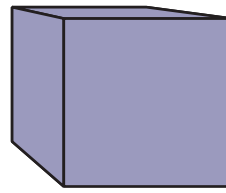
a.

3 inches



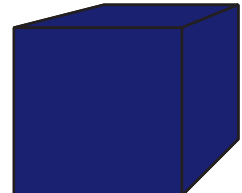
b.

1.5 ft



c.

13 cm



.....

4. Lucia is varnishing a 6 ft by 6 ft fence.

If a tin of varnish can cover 36 ft², how many tins of varnish will she need?

5. Mario wants to plant some basil in a cube-shaped planter box.

If one side of the box measures 4.5 inches, how much soil will be needed to fill the planter?

.....

Example

Write the number 146,000,000 in scientific notation.

Solution

$$146,000,000 = 1.46 \times 100,000,000 = 1.46 \times 10^8$$

Write it as a product of two factors —
one factor smaller than 10
and the other a power of 10.

6. Write each number in scientific notation.

a. 86,000

b. -9,000,000

c. 85,500,000,000,000,000

7. The following numbers are written in scientific notation. Write each number out in full.

a. -6.03×10^5

b. 7.8×10^{10}

c. 9.004×10^8

8. The world population in July 2006 was approximately 6.55×10^9 .

Write this number out in full.

9. An internet company lost \$48,000,000 last quarter.

Write this amount in scientific notation.

10. 500,000 people were asked if they believed in ghosts, and 128,000 people said they did.

How many people asked didn't say they believed in ghosts? Write your answer in scientific notation.

Lesson
2.5.4

More on the Order of Operations

California Standards: Number Sense 1.2

Example

Evaluate $3 + 8 \times (4 + 1)^2 - 15 \div 3$

Solution

$$3 + 8 \times (4 + 1)^2 - 15 \div 3$$

$$= 3 + 8 \times (5)^2 - 15 \div 3$$

$$= 3 + 8 \times 25 - 15 \div 3$$

$$= 3 + 200 - 5$$

$$= 198$$

Do the sum in the parentheses

Evaluate the exponent

Do the multiplication and then the division

Finally do the addition and subtraction

1. Evaluate the expression: $15 + 2 \times 3^3$

2. Evaluate the expression: $(6 - 9)^2 + 8^2 \div 2$

3. Evaluate the expression: $(-1)^2 + (-9)^2 \times 2$

4. Evaluate the expression: $(3 \times 4 - 3)^3 + (9 \div 3)^2 \times 7$

5. Simplify the following expressions.

a. $\left(\frac{4}{5} - \frac{3}{10}\right)^2$

b. $\left(\frac{3}{5} \cdot \frac{1}{5} + \frac{4}{5} \cdot \frac{2}{5}\right)^3$

c. $\left[\frac{1}{2} - \left(-\frac{2}{3}\right)^2\right] + \left[-\left(\frac{2}{3}\right)^2 - \frac{4}{5}\right]$

d. $\left(3\frac{1}{2}\right)^2 - \left(1\frac{1}{3}\right)^3$

.....

6. Simplify.

a. $1.65 - 2.8 \div (-0.2) + 0.4$

b. $(8.3)^2 - [(0.02)(12) + 8.5]$

.....

.....

c. $[8.3 - 2(3)]^2 - [4^2 - 2^4] - [3.3(2) - 2.8(4)]$

d. $(-0.1)^3 + (-0.1)^3 + (-0.1)^3$

.....

.....

7. When asked to simplify the expression $3 + (2.4)^2 \times (0.1 + 8.6)$, Courtney said the answer was exactly 53.11. Is Courtney correct? Show your working.

8. Simplify the expression $\left(\frac{1}{4} \div \frac{1}{2}\right)^2 + 3 \cdot \frac{6}{20}$. Show your working.

9. In the expression $(z + x^9 - x + (-y^3))^2$, x , y , and z stand for whole numbers. If you evaluate the expression, will it have a positive or negative value? Explain your answer.

10. Simplify: $(0.3 + 0.6 \cdot 8)^2 - \left(\frac{1}{2}\right)^2 \cdot 4$ -----

11. Simplify: $\left(\frac{3}{5}\right)^2 - \left(2.4\left(\frac{3}{5}\right) - 0.3\left(\frac{8}{5}\right)\right)$. Express the answer as a fraction.

12. The length (in inches) of a particular car can be found using the formula:

$$3w - \left(\frac{h}{11}\right)^2 \text{ where } h \text{ equals the height of the car in inches and } w \text{ equals the width of the car in inches.}$$

If the width of a car is 74.1 inches, and the height is 58.3 inches, find the car's length to the nearest inch.

Lesson
2.6.1

Perfect Squares and Their Roots

California Standards: Number Sense 2.4

Example

Decide whether the following numbers are perfect squares. a. 72 b. 100

Solution

a. **no**, no integer multiplied by itself gives the answer 72.

b. **yes**, $10 \times 10 = 100$

A perfect square is the product of an integer and itself.

1. Decide whether the following numbers are perfect squares.

a. 225

b. 34

c. 32

d. 49

e. 10,000

2. Is -81 a perfect square number? Explain your answer.

3. Tick the correct box to show whether each statement is true or false.

	True	False
a. The square root of 121 is either 12 or -12 .	<input type="checkbox"/>	<input type="checkbox"/>
b. 9 squared is 81.	<input type="checkbox"/>	<input type="checkbox"/>
c. 400 is a perfect square number.	<input type="checkbox"/>	<input type="checkbox"/>
d. 80 is a perfect square number.	<input type="checkbox"/>	<input type="checkbox"/>
e. The square root of 36 is either 6 or -6 .	<input type="checkbox"/>	<input type="checkbox"/>

4. How are square numbers and square roots related?

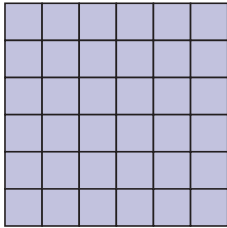
5. a. Why is $\sqrt{27}$ not a perfect square? -----

b. Why is 25 a perfect square? -----

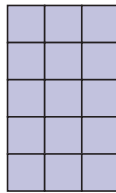
6. a. The area of a square is 16 units². Find its side length.
 b. The area of a rectangle is 8 units². Explain why 8 is not a perfect square number.

7. Diego draws the following diagrams, where the area of each small square is 1 unit². State whether the area is a perfect square number for each of the diagrams.

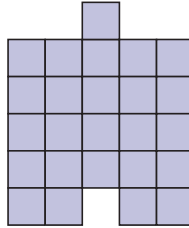
a.



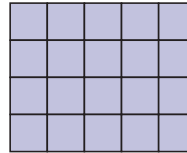
b.



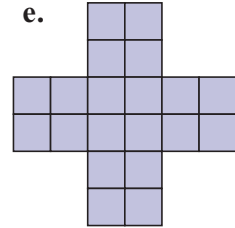
c.



d.



e.



8. Evaluate the following square roots.

a. $\sqrt{1}$

b. $\sqrt{25}$

c. $-\sqrt{81}$

d. $-\sqrt{121}$

9. Evaluate the following square roots.

a. $\sqrt{169}$

b. $-\sqrt{441}$

c. $\sqrt{324}$

d. $-\sqrt{625}$

10. Evaluate the following expressions.

a. $9^{\frac{1}{2}}$

b. $64^{\frac{1}{2}}$

c. $196^{\frac{1}{2}}$

d. $-289^{\frac{1}{2}}$

11. Pilar has a square playpen that has an area of 81 ft². She wants to increase the side length of the pen by 3 feet. How much will the area of the playpen increase by?

Lesson
2.6.2

Irrational Numbers

California Standards: Number Sense 1.4

1. Write each of the following rational numbers as a fraction in its simplest form.

a. 0.5

b. 19

c. $\sqrt{25}$

d. $0.\bar{6}$

.....

.....

.....

.....

2. Write the following rational numbers as fractions in their simplest form.

a. $\sqrt{576}$

b. $\sqrt{0.81}$

c. 7.05

d. -0.4

.....

.....

.....

.....

Example

Mary says that the number 1.3527389... is irrational. Tom says it's not possible to be sure whether the number is rational or irrational. Who is right?

Solution

Tom is right — the digits may start to repeat eventually but there aren't enough decimal places given to be sure either way.

3. Identify each number as rational or irrational. Explain your answer.

a. $\sqrt{100}$

.....

b. $\sqrt{18}$

.....

c. $\frac{5}{6}$

.....

4. Explain why 0.435435435... is a rational number but 0.435745378... seems to be irrational.

.....

5. a. Write down a rational number with a decimal that terminates.

b. Write down a repeating decimal, greater than the number written in part a.

c. Write an irrational number that lies in between your answers for a. and b.

6. Find one value for x that will make the expression $\frac{\sqrt{2}}{x}$ a rational number.

.....

7. Decide whether the following statements are true or false.

True *False*

a. All integers are rational numbers.

☐
☐

b. Some irrational numbers are also rational numbers.

☐
☐

c. It is easy to prove that 4.35627109346... is an irrational number.

☐
☐

8. Complete the table below.

Number	Rational/Irrational	Reason
$\sqrt{26}$		
0.1		
$18\frac{1}{3}$		
	Rational	Terminating decimal / Can be written as $\frac{5}{2}$
	Irrational	32 is not a perfect square
	Rational	Repeating decimal

9. a. Give a rational number between $\sqrt{29}$ and $\sqrt{45}$

.....

b. Give an irrational number between 8 and 12.

.....

c. Write down two rational numbers and two irrational numbers between 6 and 7.

.....

10. State whether the following numbers are rational or irrational.

a. $0.232343\overline{67}$

b. 9π

11. Vicente said, “The number 2.828427125... is definitely a rational number because it is the square root of an integer, 8.” Is he right? Explain your answer.

.....

.....

12. Is it possible to write 9.23565564595616... as a fraction? Explain your answer.

.....

Lesson
2.6.3

Estimating Irrational Roots

California Standards: Number Sense 2.4, Mathematical Reasoning 2.7

1. State whether each of the following is rational or irrational.

a. $\sqrt{5}$

.....

b. $\sqrt{324}$

.....

c. $\sqrt{11}$

.....

2. Decide whether the following statements are true or false.

True

False

a. The square root of 48 is between 8 and 9.

☐
☐

b. The square root of 34 is between 4 and 5.

☐
☐

c. The square root of 69 is closer to 8 than 9.

☐
☐

d. The square root of 130 is between 11 and 12.

☐
☐

3. Use your calculator to approximate the following square roots to 1 decimal place.

a. $\sqrt{22}$

.....

b. $\sqrt{55}$

.....

c. $\sqrt{91}$

.....

d. $\sqrt{18}$

.....

e. $\sqrt{48}$

.....

4. Use your calculator to approximate the following square roots to 3 decimal places.

a. $\sqrt{0.0008}$

b. $\sqrt{0.077}$

5. Approximate the side length of a square with an area of 38 ft² to 1 decimal place.

.....

6. Square 1 has an area of 9.0 m², and square 2 has an area of 6.0 m².

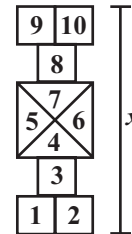
Approximate the difference between the side lengths of these two squares to 1 decimal place.

.....

7. Mr. Lopez is planning a square patio addition to his house. The area of the patio will be 110 ft^2 .
Approximate the length and width of the patio to 1 decimal place.

8. A square picture frame has an area of 176.3 cm^2 .
Find the side length of the frame to 1 decimal place.

9. Eric is painting a hopscotch grid in the playground of his local elementary school, as shown. Each small square has an area of 182 inches^2 and the large center square has an area of 728 inches^2 .

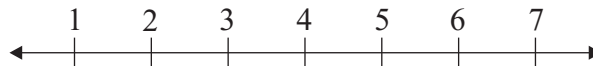


Estimate the length, x , of the hopscotch grid to the nearest inch.

Example

Find the numbers that the following roots lie between on the number line shown.

a. $\sqrt{11}$ b. $\sqrt{26}$



Solution

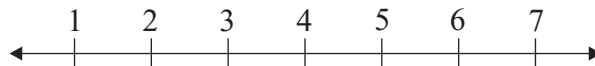
a. $\sqrt{9} = 3, \sqrt{16} = 4$, so $\sqrt{11}$ lies between 3 and 4 on the number line.

b. $\sqrt{25} = 5, \sqrt{36} = 6$, so $\sqrt{26}$ lies between 5 and 6 on the number line.

10. Find the numbers that the following roots lie between on the number line shown.

a. $\sqrt{5}$

b. $\sqrt{42}$



11. Find the whole numbers that each of the following roots lie between.

a. $\sqrt{167}$

b. $\sqrt{65}$

c. $\sqrt{219}$

.....

12. Jo is making a child's toy where kids push different shaped blocks into matching holes.

She has made a square hole with a side length of 5 cm for a cube with a surface area of 180 cm^2 .

Will the cube fit through the hole? Use estimation and explain your answer.

.....
.....

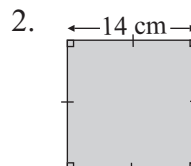
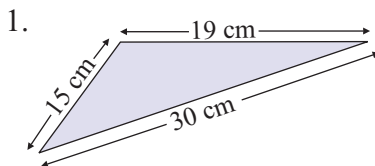
Lesson
3.1.1

Polygons and Perimeter

California Standards: Measurement and Geometry 2.1

Example

Find the perimeter of each of the following figures.



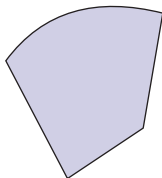
These marks tell you that the sides are equal in length.

Solution

1. Perimeter = sum of the lengths of the sides = $15\text{ cm} + 19\text{ cm} + 30\text{ cm} = 64\text{ cm}$.
2. Perimeter = $4 \times \text{length of side} = 4 \times 14\text{ cm} = 56\text{ cm}$.

1. Explain why each of the following figures is not a polygon.

a.



b.

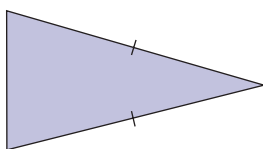


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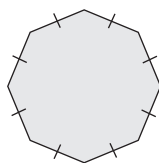
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2. Identify each of the following polygons as either regular or irregular.

a.



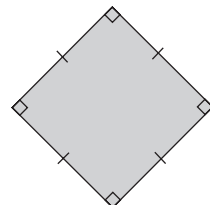
b.



c.



d.



.....

.....

.....

.....

3. Find the perimeter of the following shapes.

a. An equilateral triangle with side length of 33 inches.

b. A rectangle with side lengths of 19 inches and 12 inches.

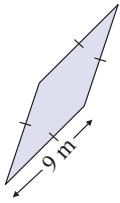
c. A regular octagon with side length of 10 centimeters.

4. A rectangular picture measures 26 inches by 16 inches. What is its perimeter?

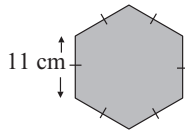
.....

5. Find the perimeter of each of the shapes below.

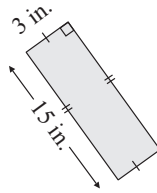
a.



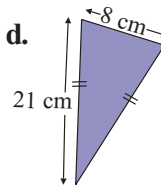
b.



c.



d.

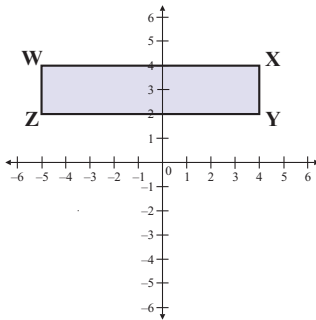


6. a. The perimeter of an equilateral triangle is 24 cm. Find the length of each side.

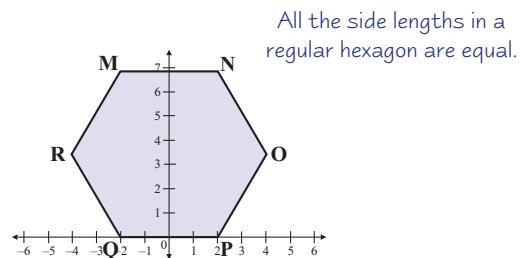
b. The perimeter of a regular pentagon is 45 inches. Find the length of each side.

c. The perimeter of a rectangle is 24 meters. The length of the rectangle is 8 meters. Find the width of the rectangle.

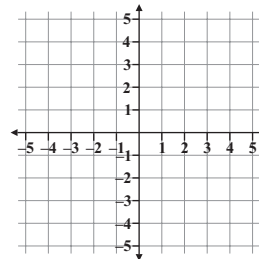
7. a. Find the perimeter of rectangle **WXYZ**.



b. Find the perimeter of regular hexagon **MNOPQR**.

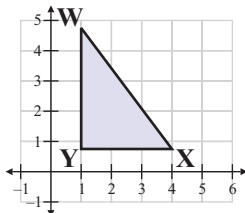


8. The points **A**(-4, 2), **B**(-2, 2), **C**(-2, -4), and **D**(-4, -4) form rectangle **ABCD**. Find its perimeter.

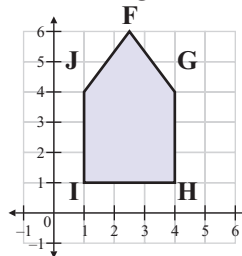


Plot the points to determine the perimeter.

9. a. The perimeter of triangle **WXY** is 12 units. Find the length of all three sides.



b. The perimeter of pentagon **FGHIJ** is 14 units. Find the length of each side.



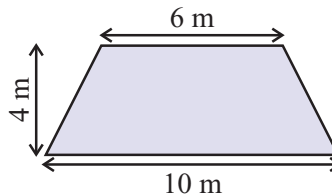
Lesson
3.1.2

Areas of Polygons

California Standards: Measurement and Geometry 2.1

Example

Find the area of the trapezoid in the diagram.

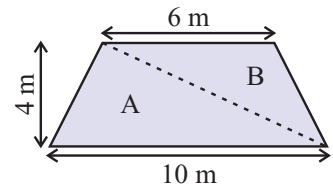


Solution

Split the trapezoid into two triangles and work out the area of each.

$$A: \text{area} = \frac{1}{2}bh = \frac{1}{2} \times 10 \times 4 = 20 \text{ m}^2 \quad B: \text{area} = \frac{1}{2}bh = \frac{1}{2} \times 6 \times 4 = 12 \text{ m}^2$$

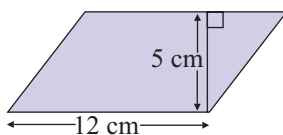
$$\text{Total area} = A + B = 20 + 12 = 32 \text{ m}^2$$



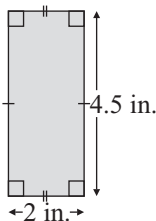
1. Find the area of the following shapes.

First, identify each figure and work out which equation you need to use to find the area.

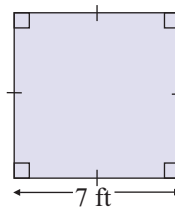
a.



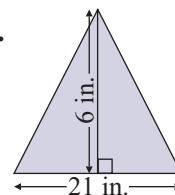
b.



c.



d.

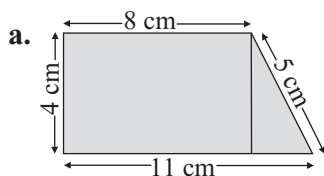


2. a. Find the area of a square with side length of 20 inches.
- b. Find the area of a rectangle with side lengths of 5 inches and 8 inches.
- c. Find the area of a triangle with a base of 10 cm and a height of 16 cm.
- d. Find the area of a parallelogram with a height of 7 yards and a base of 13 yards.
3. The perimeter of a rectangle is 18 inches.
- a. List the four possible combinations of length and width of the rectangle if each dimension is an integer number of inches
-
- b. Find the area of each possible rectangle from part a.
-
4. a. Find the area of a square with side length of 90 centimeters.
- b. Find the area of a parallelogram with a base of $\frac{1}{2}$ yard and a height of $\frac{1}{4}$ yard.
- c. Find the area of a triangle with a base of 16 inches and a height of 12 inches.

5. A square has a perimeter of 52 centimeters. Find its area.

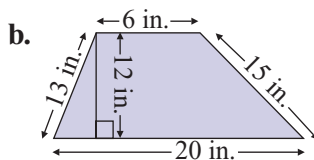
.....

6. Find the perimeter and area of each of the following trapezoids.



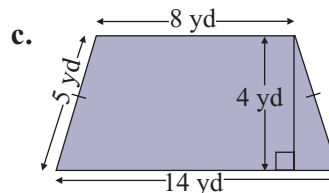
Perimeter:

Area:



Perimeter:

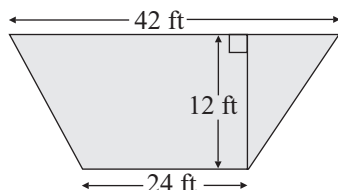
Area:



Perimeter:

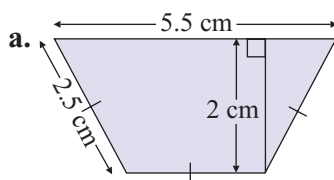
Area:

7. The picture below shows the stage at a community hall. Find the area of the stage.



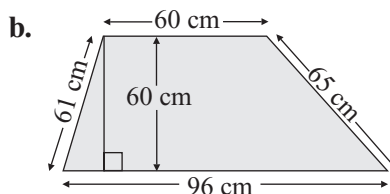
.....

8. Find the perimeter and area of each of the following trapezoids.



Perimeter:

Area:



Perimeter:

Area:

9. The height of a trapezoid is 15 cm and the length of its base is 7 cm.
If the area of the trapezoid is 127.5 cm^2 , what is the length of the side parallel to the base?

.....

10. The area of a trapezoid is 66 square inches. The length of its parallel edges are 4 inches and 7 inches. Find the height of the trapezoid.

.....

Lesson
3.1.3

Circles

California Standards: Measurement and Geometry 2.1

Example

Find the circumference of a circle with a radius of 19 meters. Express your answer in terms of π .

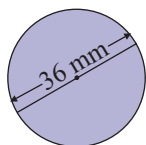
Solution

Circumference, $C = 2\pi r = 2\pi \times 19 = 38\pi$ m.

1. a. The diameter of a circle is 120 mm. Find the length of the radius.
- b. The radius of a circle is 50 cm. Find the length of the diameter.
- c. The radius of a circle is $14x$. Find the length of the diameter.
- d. The diameter of a circle is $8x + 6$. Find the length of the radius.

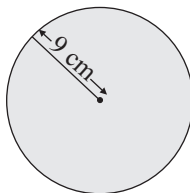
2. Find the specified dimension for each of the following circles.

a.



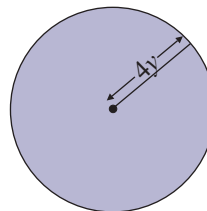
$r =$

b.



$d =$

c.



$d =$

3. Why is a circle not considered a polygon?

.....

4. Find the circumference of the following circles. Express your answer in terms of π .

a. A circle with radius 16 cm.

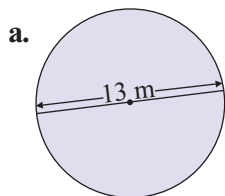
b. A circle with radius 4.2 inches.

c. A circle with diameter 1.5 ft.

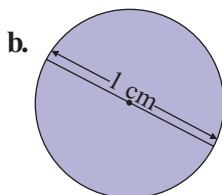
5. The circumference of a circle is 29π inches. Find the length of the radius.

.....

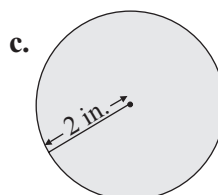
6. Find the circumference of the following circles. Use $\pi = 3.14$ in your calculations.



.....

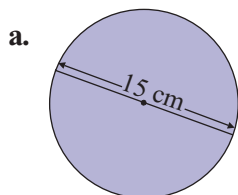


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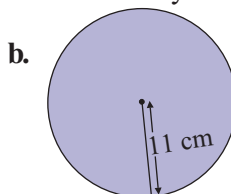


.....

7. Find the circumference of the following circles. Use $\pi = 3.14$ in your calculations.

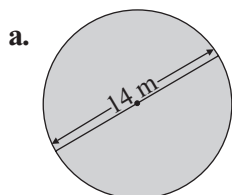


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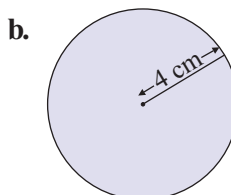


.....

8. Find the area of the following circles. Leave your answer in terms of π .

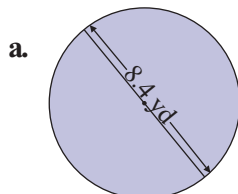


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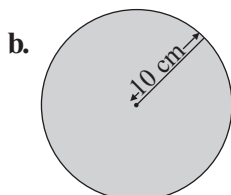


.....

9. Find the area of the following circles. Use $\pi = 3.14$ in your calculations. Round each answer to the nearest hundredth.



.....



.....

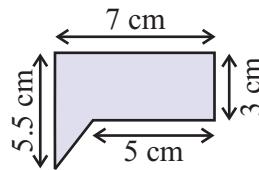
Lesson
3.1.4

Areas of Complex Shapes

California Standards: Measurement and Geometry 2.2

Example

Find the area of the figure in the diagram.



Solution

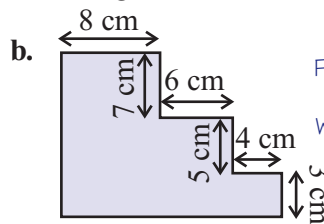
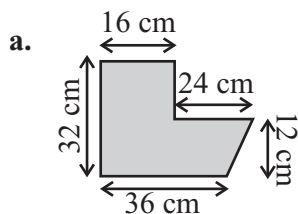
Split the figure into a rectangle and a triangle, and find the area of each separately.

$$\text{Area of rectangle} = 7 \times 3 = 21 \text{ cm}^2$$

$$\text{Area of triangle} = \frac{1}{2} (7 - 5)(5.5 - 3) = \frac{1}{2} \times 2 \times 2.5 = 2.5 \text{ cm}^2$$

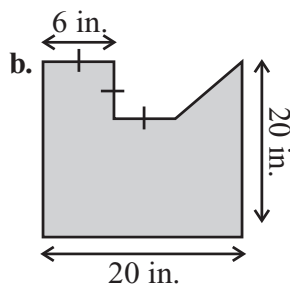
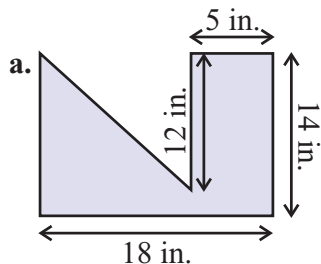
$$\text{Total area} = \text{area of rectangle} + \text{area of triangle} = 21 + 2.5 = 23.5 \text{ cm}^2$$

1. Find the area of each of the following irregular figures using the sum of areas method.



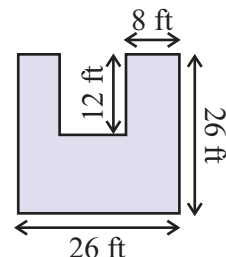
Find the total area of a shape by splitting it into simpler parts. Work out the areas of each part, then add them together.

2. Find the area of each of the following irregular figures by using the difference of areas method.

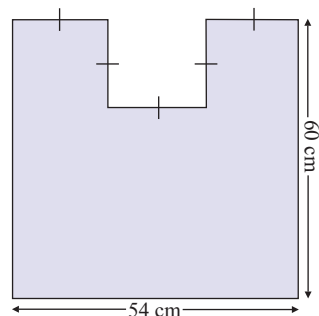


Treat the shape as one large simple shape, with a chunk taken out of it. Work out the area of the "chunk" and subtract it from the area of the larger shape.

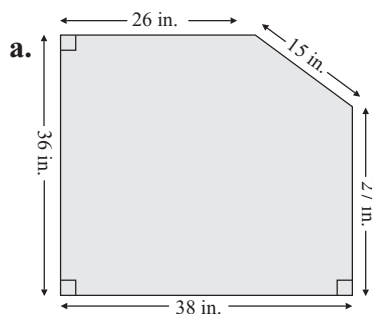
3. Explain why it is not possible to find the area of this figure.



4. The figure shown has a perimeter of 264 centimeters.
Find the area of the figure.

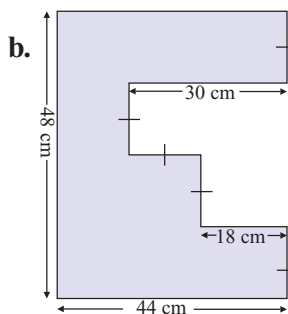


5. Find the perimeter and area of each of the following irregular figures.



Perimeter:

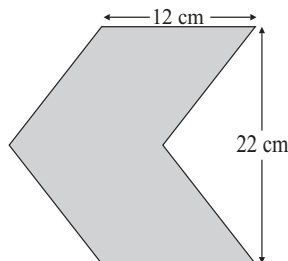
Area:



Perimeter:

Area:

6. a. Find the area of the irregular polygon in the diagram using the sum of areas method.

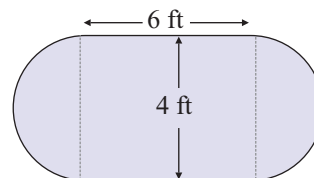


.....
.....

- b. Briefly describe another method you could use to find the area of the polygon in part a.

.....
.....

7. Devon built a mini-racetrack for his son's toy cars, as shown in the diagram. Find the area of the region bounded by the track. Use $\pi = 3.14$ in your calculations.



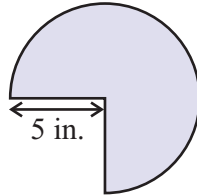
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Lesson
3.1.5

More Complex Shapes

California Standards: Measurement and Geometry 2.2

1. Find the area of the shape in the diagram. Use $\pi = 3.14$ in your calculations.



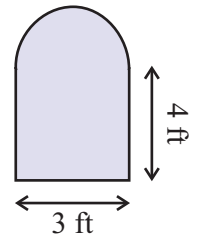
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2. Katherine needs to buy fabric to make a blind for the window shown in the diagram. Determine the area of the window. Use $\pi = 3.14$ in your calculations.

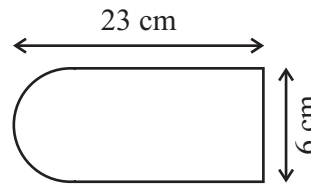
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Example

Find the perimeter of the shape in the diagram.



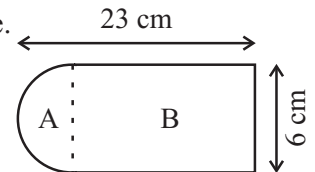
Solution

Split the shape into two simpler shapes — A, a semicircle and B, a rectangle. Find the part of the perimeter of each shape shared with the original shape:

$$A: p = \frac{1}{2}(2\pi r), \text{ where } r = \frac{6}{2} = 3. \quad p = \frac{1}{2}(2 \times \pi \times 3) = 3\pi \approx 9.42 \text{ cm}$$

$$B: p = 2l + w, \text{ where } l = 23 - 3 = 20 \text{ and } w = 6. \quad p = 2 \times 20 + 6 = 46 \text{ cm}$$

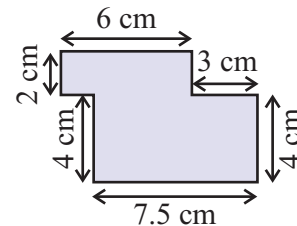
The perimeter of the original shape is $9.42 + 46 = 55.42 \text{ cm}$.



3. Find the perimeter of the shape in the diagram.

.....

.....

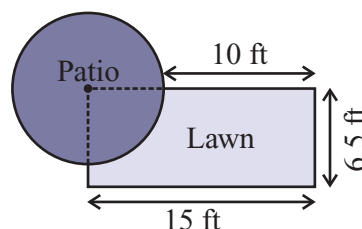


4. Lori baked a circular cake with a diameter of 14 in. for her office party. The cake was cut into eight equal slices for the people in Lori's office. Determine the area and perimeter of the base of each slice of cake. Use $\pi = 3.14$ in your calculations.

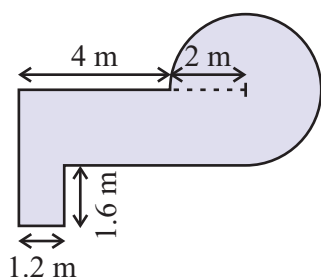
.....

5. Find the area of a circular mirror frame if the diameter of the frame is 3 ft and the diameter of the circular mirror inside the frame is 1.5 ft. Use $\pi = 3.14$ in your calculations.

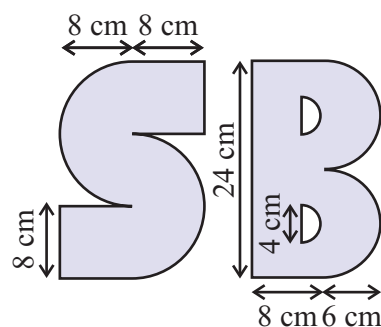
6. Anthea is remodeling her garden. The diagram shows the areas of lawn and patio as they are now. Anthea is going to returf the whole area and put a border around the edge of it. Find the total area of the lawn and patio and the distance around the edge of the combined area. Use $\pi = 3.14$ in your calculations.



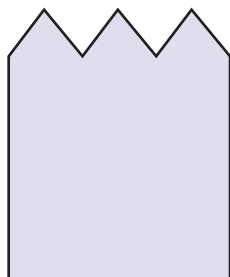
7. Find the area and perimeter of the shape shown in the diagram. Use $\pi = 3.14$ in your calculations.



8. Susie has made a stencil of her initials, using a combination of rectangles and semicircles as shown in the diagram. Determine the total perimeter and area of the two letters. Use $\pi = 3.14$ in your calculations.



9. Describe how you would break up the shape in the diagram if asked to find its area and perimeter.



Lesson 3.2.1

Plotting Points

California Standards: Measurement and Geometry 3.2

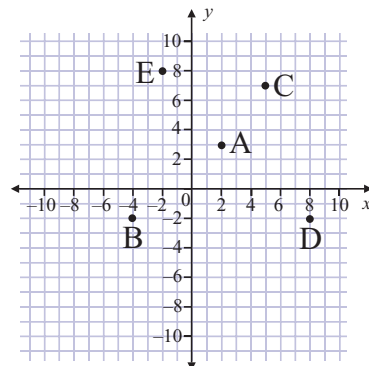
Example

Identify each of the following points and their quadrants.

- $(5, 7)$
- $(-4, -2)$
- $(8, -2)$

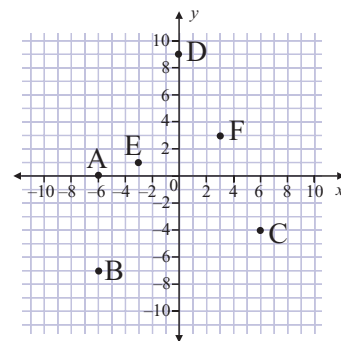
Solution

- C, Quadrant I
- B, Quadrant III
- D, Quadrant IV



- Identify the points with the following coordinates.

- | | | | |
|---------------|-------|--------------|-------|
| a. $(0, 9)$ | | b. $(-3, 1)$ | |
| c. $(3, 3)$ | | d. $(-6, 0)$ | |
| e. $(-6, -7)$ | | f. $(6, -4)$ | |



- Determine whether each of the following statements is true or false.

- An ordered pair is written (y, x) .
- The origin is the intersection of the x -axis and y -axis.
- Points in quadrant III are of the form $(+, -)$.
- Point $(7, 5)$ is in quadrant I.
- The y -axis is the horizontal line on the coordinate plane.
- Point $(7, 0)$ lies on the x -axis.

.....

.....

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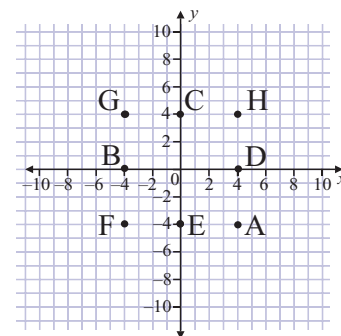
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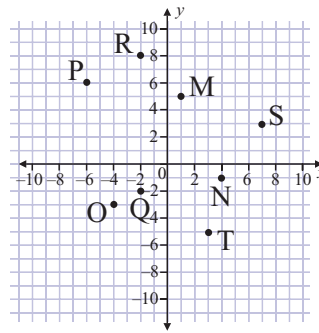
- Identify the points with the following coordinates.

- | | | | |
|--------------|-------|---------------|-------|
| a. $(-4, 4)$ | | b. $(4, -4)$ | |
| c. $(-4, 0)$ | | d. $(-4, -4)$ | |
| e. $(0, -4)$ | | f. $(4, 0)$ | |



4. Identify the quadrant containing each point.

- | | |
|------------|------------|
| a. Q | b. N |
| c. R | d. O |
| e. M | f. T |
| g. S | h. P |

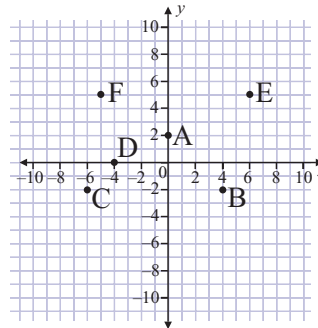


5. Complete each sentence to make a true statement.

- The x -coordinate of the point $(-3, 7)$ is
- The orientation of the y -axis is
- The point $(-4, 7)$ lies in quadrant
- The is the intersection of the x -axis and y -axis.
- The point is in quadrant IV.

6. Identify the ordered pair for each point.

- | | |
|------------|------------|
| a. A | b. B |
| c. C | d. D |
| e. E | f. F |



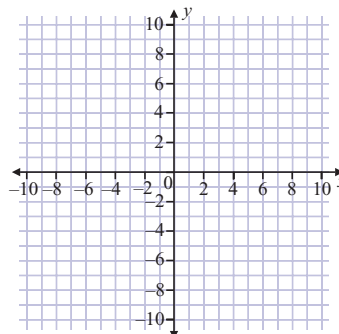
From the origin the x -coordinate moves the point horizontally and the y -coordinate moves the point vertically.

7. Write a description of how to plot each point.

- $(8, 3)$
- $(-5, -3)$
- $(8, 0)$

8. Plot and label each of the following points.

- | | | |
|---------------|--------------|--------------|
| a. $(4, -7)$ | b. $(0, -6)$ | c. $(3, 5)$ |
| d. $(-6, -6)$ | e. $(0, 0)$ | f. $(-5, 3)$ |



Lesson
3.2.2

Drawing Shapes in the Coordinate Plane

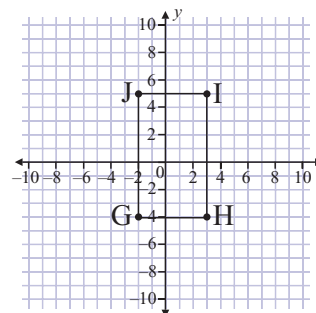
California Standards: Measurement and Geometry 3.2

Example

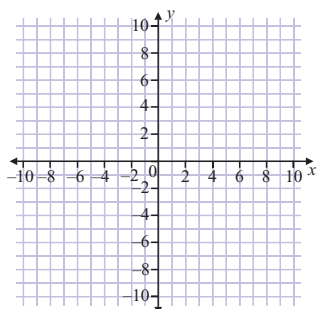
- Plot the points $G(-2, -4)$, $H(3, -4)$, $I(3, 5)$.
- Identify the missing ordered pair to form rectangle $GHIJ$.
- Plot the point J .
- Graph the rectangle $GHIJ$.

Solution

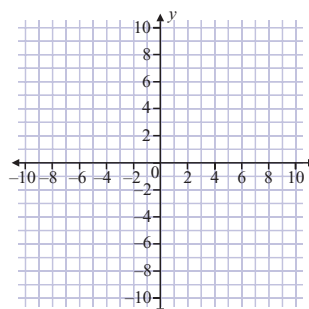
- See diagram
- $(-2, 5)$
- See diagram
- See diagram



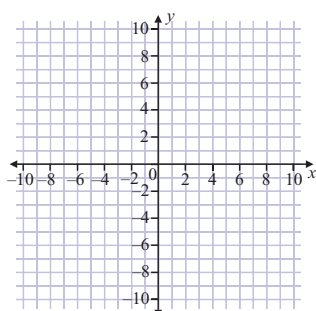
- Graph triangle BCD .
 $B(-4, 0)$ $C(5, 7)$ $D(4, 1)$



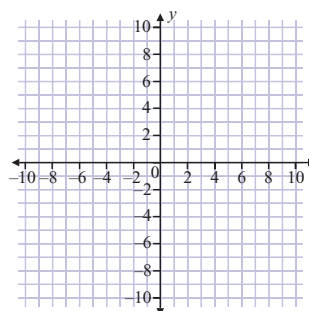
- Graph trapezoid $PQRS$.
 $P(-7, 4)$ $Q(-2, 4)$ $R(6, -4)$ $S(-5, -4)$



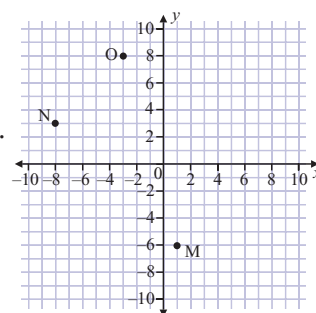
- Graph and classify the following two polygons.
 - $W(-5, -1)$ $X(2, 3)$ $Y(6, -4)$ $Z(-1, -8)$



- $R(-6, 2)$ $S(-4, 2)$ $T(-4, -6)$ $U(-6, -6)$

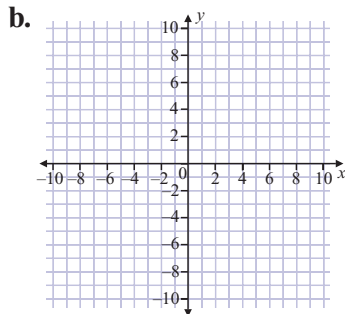
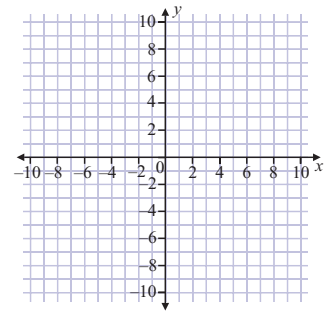


- Tatum is sketching her rectangular garden on the coordinate plane shown. She has already plotted three points, M , N , and O . Locate the fourth point and give its coordinates.



4. a. Rectangle ABCD contains the points A(-6, 3), B(1, 5), and D(-5, -1). Plot the points A, B, and D, then identify and plot point C.

Point C:



Parallelogram RSTU contains the points R(-5, 5), S(1, 5), and U(-3, 0). Plot the points R, S, and U, then identify and plot point T.

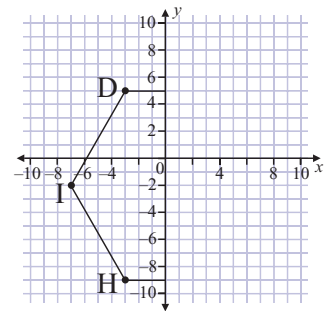
Point T:

5. Hexagon DEFGHI is symmetrical about the y -axis.

a. Complete the hexagon.

b. Identify the ordered pairs that complete the hexagon.

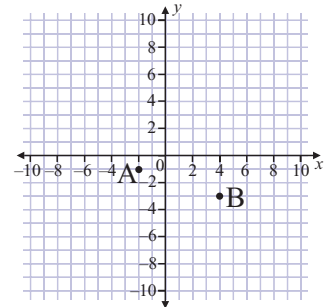
.....,,



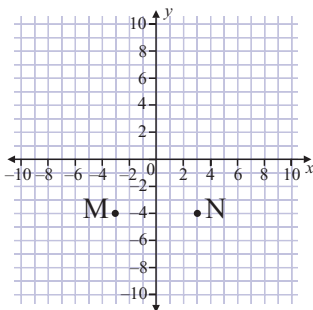
6. In how many ways could square ABCD be completed? Explain your answer. Identify the possible ordered pairs that would complete the square.

.....

.....



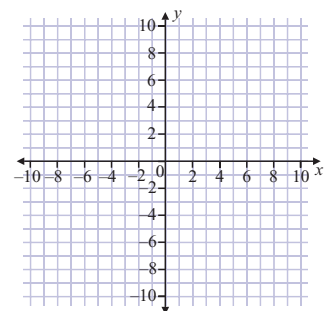
7. Explain why triangle MNP will be isosceles if P is on the y -axis and not at (0, -4).



.....

.....

8. Graph a circle with a radius of 4 units and a center at (2, -1).



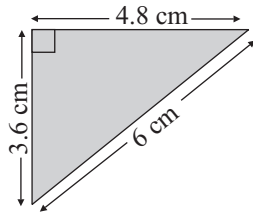
Lesson
3.3.1

The Pythagorean Theorem

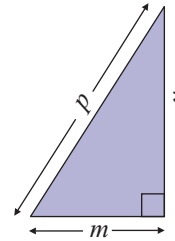
California Standards: Measurement and Geometry 3.3

1. Identify the hypotenuse on each of the following triangles.

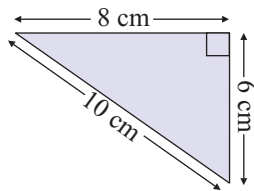
a.



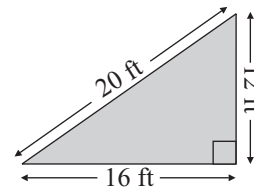
b.



c.

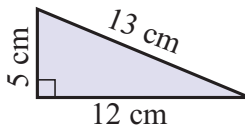


d.

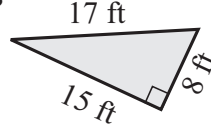


2. Which of the following triangles cannot be used to verify the Pythagorean theorem?

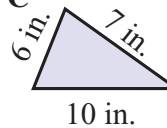
A



B



C



Example

Verify the Pythagorean theorem for a right triangle with side lengths $a = 12$, $b = 16$, and $c = 20$.

Solution

The Pythagorean theorem states that $a^2 + b^2 = c^2$.

$$12^2 + 16^2 = 20^2 \quad \text{Substitute the side lengths into the equation.}$$

$$144 + 256 = 400 \quad \text{Simplify both sides.}$$

$400 = 400$ so the Pythagorean theorem is verified in this case.

3. Verify the Pythagorean theorem for a right triangle with side lengths $a = 30$, $b = 16$, and $c = 34$.

Lesson

3.3.2 Using the Pythagorean Theorem

California Standards: Measurement and Geometry 3.2, 3.3

Example

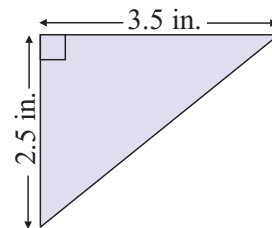
Find the length of the hypotenuse of the triangle in the diagram.

SolutionThe Pythagorean theorem states that $a^2 + b^2 = c^2$.

$$c^2 = 3.5^2 + 2.5^2 \quad \text{Substitute the known side lengths.}$$

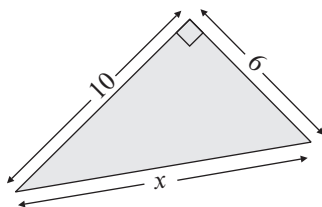
$$c^2 = 12.25 + 6.25$$

$$c = \sqrt{18.5} \approx 4.3 \text{ in.}$$



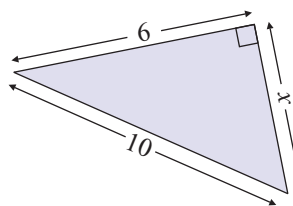
1. Write an equation for the length
- x
- in the following triangles using the Pythagorean theorem.

a.



.....

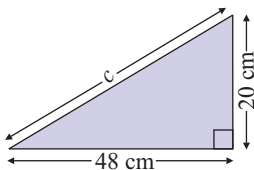
b.



.....

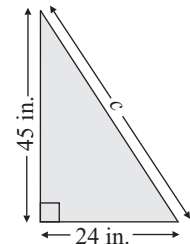
2. Find the missing side length for each of the following right triangles.

a.



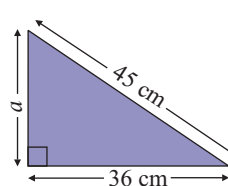
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b.



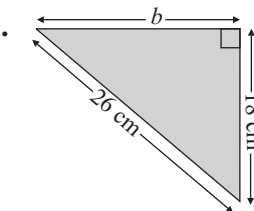
.....

c.



.....

d.



.....

3. Determine whether the following statements are true or false.

a. The leg is the longest side of a right triangle.

.....

b. For legs a and b , $a > b$ in every right triangle.

.....

c. A triangle with side lengths 10 cm, 24 cm, and 26 cm is a right triangle.

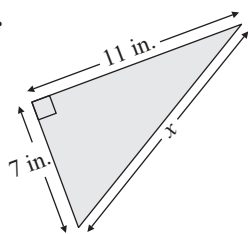
.....

d. A right triangle with sides of 48 in. 90 in. and 102 in. has a hypotenuse of 102 in.

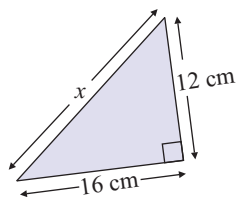
.....

4. Find the missing side length for each of the following right triangles.

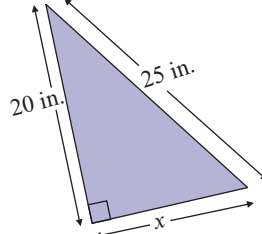
a.



b.



c.

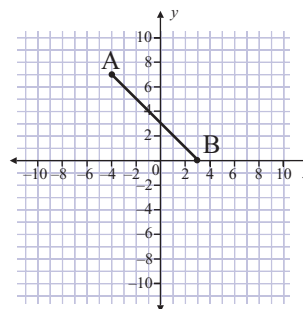


5. Find the length of the diagonal in a rectangle with a length of 30 inches and a width of 12.5 inches.

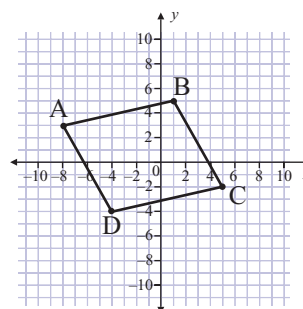
6. Rectangle ABCD has a length of 15 inches and a width of 12 inches. Estimate the length of the diagonal to the nearest whole number.

7. Find the length of each side of a square when the length of a diagonal is $24\sqrt{2}$.

8. Find the length of the line AB. Give your answer to 2 decimal places.



9. Find the perimeter of the parallelogram ABCD. Give your answer to 2 decimal places.



Lesson

3.3.3 Applications of the Pythagorean Theorem

California Standards: Measurement and Geometry 3.3

Example

A baseball diamond is a square with sides of 90 feet. Determine the shortest distance between first base and third base to the nearest tenth of a foot.

Solution

The shortest distance between two opposite corners of a square is the diagonal.

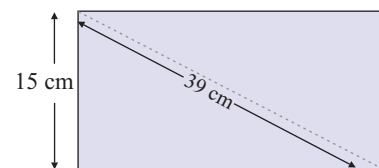
$$c^2 = 90^2 + 90^2 \quad \text{Substitute the side lengths into the Pythagorean equation.}$$

$$c^2 = 8100 + 8100$$

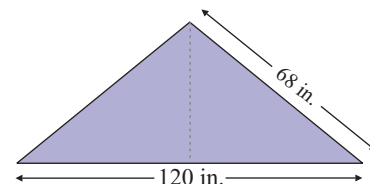
$$c = \sqrt{16,200} = 127.27922... \approx 127.3 \text{ ft (to 1 decimal place)}$$

The shortest distance from first base to third base in a baseball diamond is 127.3 ft.

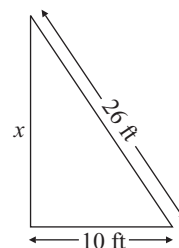
1. a. Find the area of a rectangle with a diagonal length of 39 cm and a side length of 15 cm.



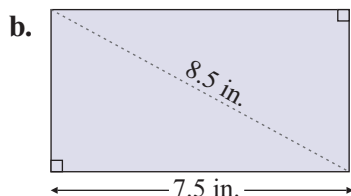
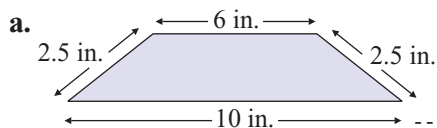
- b. Find the area of the isosceles triangle in the diagram.



2. A 26-foot ladder is placed on level ground with the bottom of the ladder 10 feet from a vertical wall. What is the height from the ground to the top of the ladder?



3. Find the area of the following polygons.



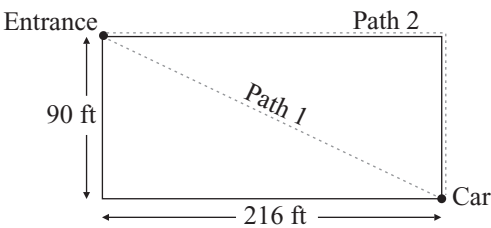
4. a. Find the height of an equilateral triangle with side lengths of 10 inches.
Give your answer to one decimal place.

.....

- b. Find the side length of a square that has a diagonal length of $\sqrt{242}$ inches.

.....

5. Mrs. Kintz parks in a corner spot in the airport parking lot, as shown below.



Mrs. Kintz can walk diagonally across the parking lot from her car to the airport entrance (Path 1), or she can walk along the sidewalk on the perimeter of the parking lot (Path 2).
How much shorter is Path 1 than Path 2?

.....

- 6 The table shows the dimensions for a guinea pig cage based on the number of guinea pigs.

Number of guinea pigs	Dimensions of cage (in.)
1	30×32
2	30×46
3	30×58
4	30×72

- a. Approximately how much longer is the diagonal measurement of a cage for two guinea pigs than the diagonal measurement of a cage for one guinea pig?

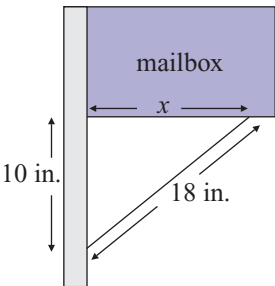
.....

- b. In the cage for four guinea pigs, how far is the centre of the floor of the cage from each of the bottom corners?

.....

7. Find the value of x in this diagram of a mailbox on a pole.
Give your answer to the nearest half an inch.

.....



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Lesson
3.3.4

Pythagorean Triples & the Converse of the Theorem

California Standards: Measurement and Geometry 3.3

1. Determine whether each set of three side lengths can be classified as a Pythagorean triple. Give a reason for each of your answers.

a. 18, 80, 82

b. 3, 7.2, 7.8

c. 10, 24, 28

2. Explain why an infinite number of Pythagorean triples can be formed from the Pythagorean triple 6, 8, 10.

.....
.....

Example

Determine whether the side lengths 15, 39, and 36 form a right triangle.

The longest side is the hypotenuse, c .

Solution

The Pythagorean theorem states that $a^2 + b^2 = c^2$. If $a^2 + b^2 = c^2$, then the three sides form a right triangle.

$$15^2 + 36^2 = 39^2 \quad \text{Substitute the side lengths into the Pythagorean equation.}$$

$$225 + 1296 = 1521 \quad \text{Simplify both sides.}$$

$$1521 = 1521, \text{ so the three sides form a right triangle.}$$

3. Mrs. Stelzer is planning a triangular vegetable garden in the corner of her yard. She drew a sketch using a triangle with sides 3 inches, 4 inches, and 5 inches. Will her garden be a right triangle?

.....

4. Determine whether each set of three sides forms a right triangle.

a. 28, 21, 35

b. 10, 12, 16

c. 82, 80, 18

d. 40, 50, 30

5. Determine whether the following triangles are right triangles.

a. A triangle with side lengths of 5.7, 9.0, 7.5.

.....

b. A triangle with side lengths of 3.15, 8.19, 7.56.

.....

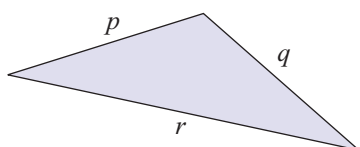
c. A triangle with side lengths of 8, 8, $8\sqrt{2}$.

.....

d. A triangle with side lengths of $5\sqrt{3}$, 8, 5.

.....

6. Explain how you could test whether this triangle is a right triangle.



.....

.....

7. Determine whether the following statements are true or false.

a. A triangle with side lengths of 4, 8, $4\sqrt{5}$ is a right triangle.

b. A triangle with side lengths of 36, 24, 40 is a right triangle.

c. A triangle with side lengths of 6.5, 6, 2.5 is a right triangle.

8. Mr. Zhang built a patio table. The table top has the following measurements: length 7.8 feet, width 10.4 feet, and diagonal length 13 feet. Did Mr. Zhang build a rectangular table top?

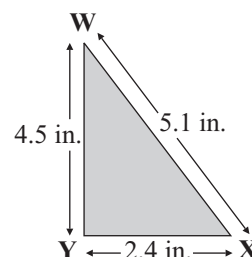
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9. Prove that triangle WXY is a right triangle.

.....

.....

.....

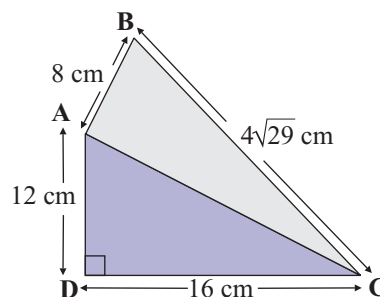


10. Prove that triangle ABC is a right triangle.

.....

.....

.....



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Lesson
3.4.1

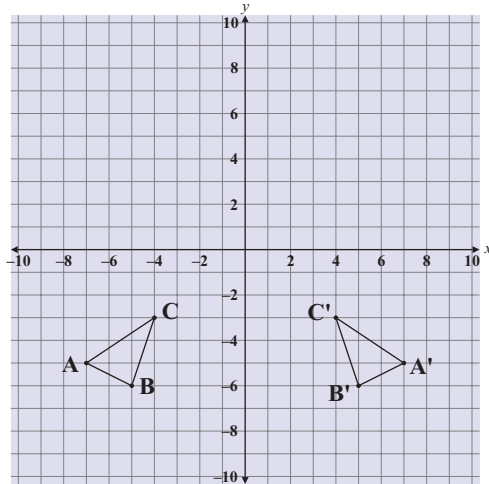
Reflections

California Standards: Measurement and Geometry 3.2

Example

Reflect triangle ABC across the y -axis. Label the image $A'B'C'$.

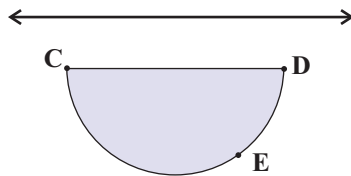
Solution



Make sure that
you label each
point correctly.

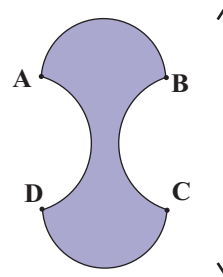
1. Reflect each figure over the given line of reflection.

a.



b.

Imagine folding the paper on the line of
reflection and tracing the figure.



2. Determine whether the following statements are true or false.

a. If a point is reflected across the x -axis its y -value will change.

.....

b. If a point is reflected across the y -axis its y -value will change.

.....

c. If a point is reflected across the y -axis, the new point will be the same distance from the y -axis.

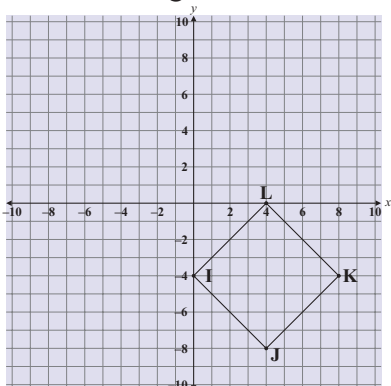
.....

3. A triangle with vertices at $A(2, 2)$, $B(5, 4)$, and $C(6, 9)$ is reflected across the x -axis.

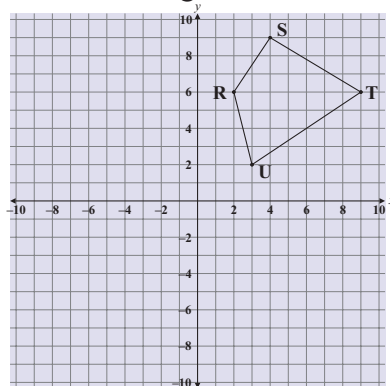
a. What happens to the y -values?

b. What happens to the x -values?

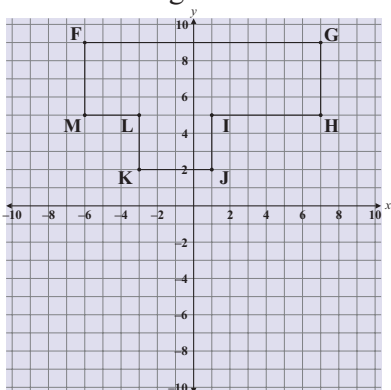
4. a. Reflect square IJKL across the y -axis.
Label the image I'J'K'L'.



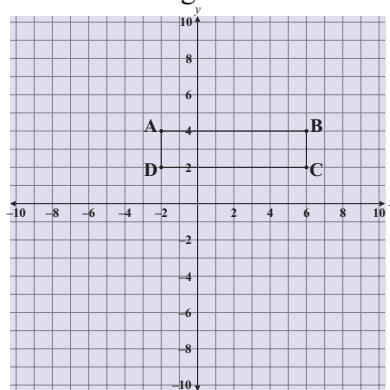
- b. Reflect quadrilateral RSTU across the y -axis.
Label the image R'S'T'U'.



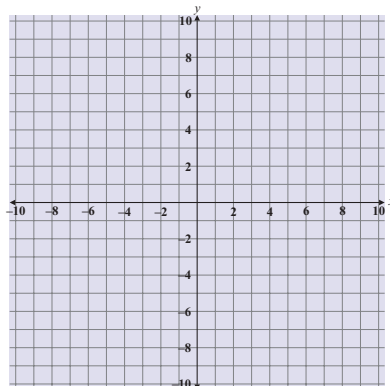
- c. Reflect octagon FGH IJKLM across the x -axis.
Label the image F'G'H'I'J'K'L'M'.



- d. Reflect rectangle ABCD across the y -axis.
Label the image A'B'C'D'.

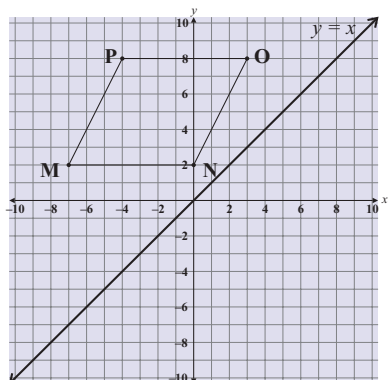


5. a. Plot the points that form quadrilateral ABCD.
A(-7, 3) B(-5, 0) C(-6, -1) D(-10, 4)
b. Reflect quadrilateral ABCD across the y -axis.
Label the image A'B'C'D'.



6. Reflect parallelogram MNOP over the line $y = x$.
Label the image M'N'O'P'.

Write the points MNOP as ordered pairs.
Switch the x -value and the y -value for
each point. Plot these four points.



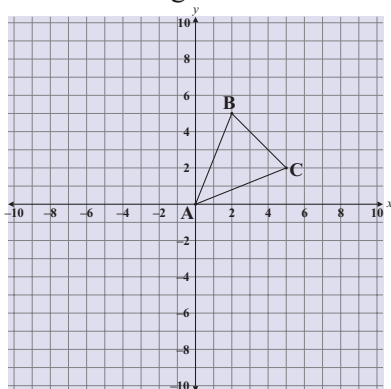
Lesson 3.4.2

Translations

California Standards: Measurement and Geometry 3.2

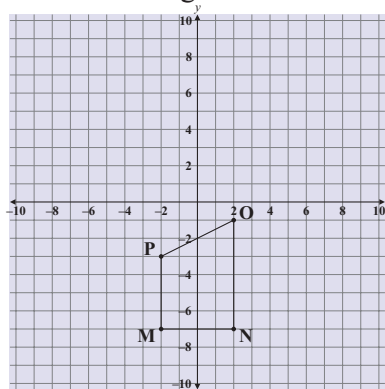
1. a. Translate triangle ABC left eight units.

Label the image A'B'C'.



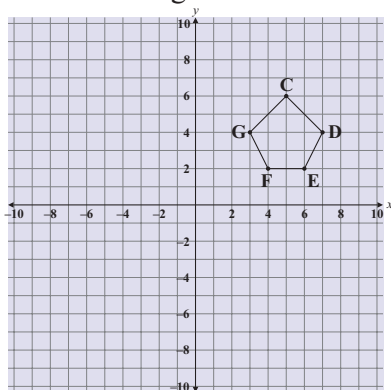
- b. Translate trapezoid MNOP up 11 units.

Label the image M'N'O'P'.



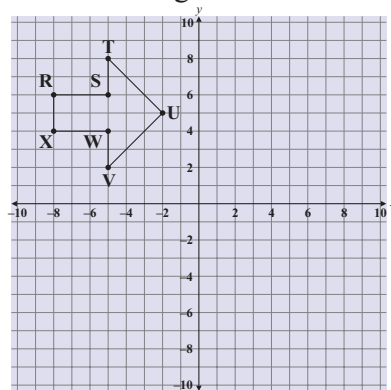
- c. Translate pentagon CDEFG down ten units.

Label the image C'D'E'F'G'.



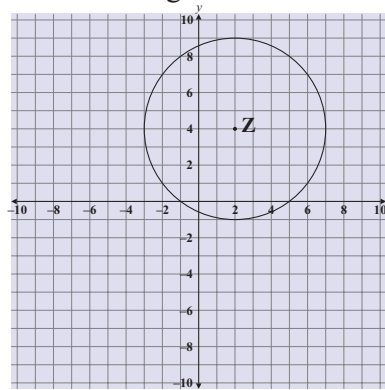
2. a. Translate figure RSTUVWX right four units.

Label the image R'S'T'U'V'W'X'.

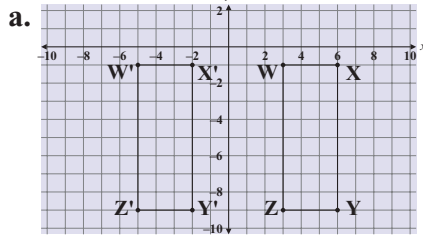


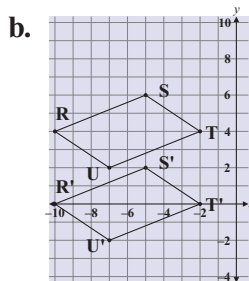
- b. Translate circle Z down six units.

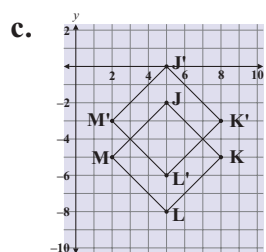
Label the image circle Z'.



3. Write a sentence to describe each translation.

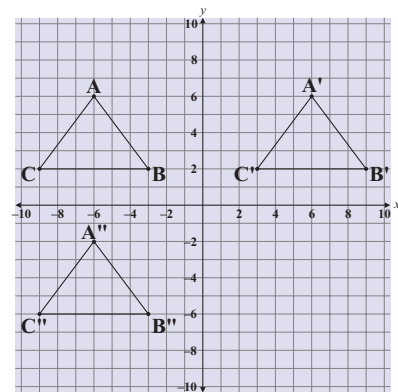






4. a. Write a sentence to describe the translation of triangle ABC to triangle A'B'C'.

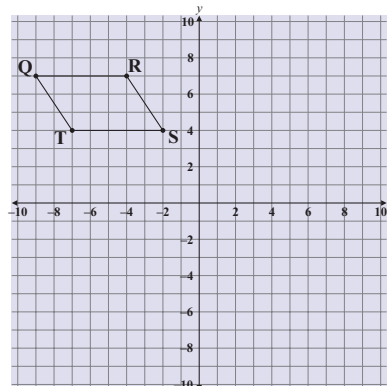
b. Write a sentence to describe the translation of triangle ABC to triangle A''B''C''.



c. In what way is triangle A'B'C' different from a reflection of triangle ABC across the y-axis?

5. a. Reflect parallelogram QRST across the x -axis. Label the image Q'R'S'T'.

b. Translate parallelogram Q'R'S'T' seven units right. Label the image Q''R''S''T''.



In other words, describe what happens to the x -coordinates and what happens to the y -coordinates.



6. Algebraically describe how to translate triangle EFG right six units.

Lesson 3.4.3

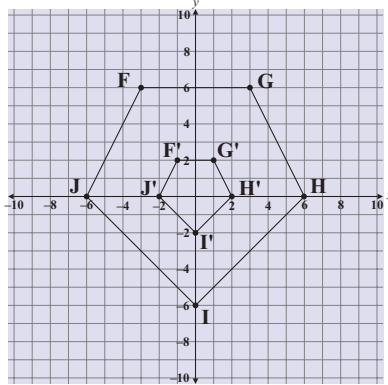
Scale Factor

California Standards: Measurement and Geometry 1.2

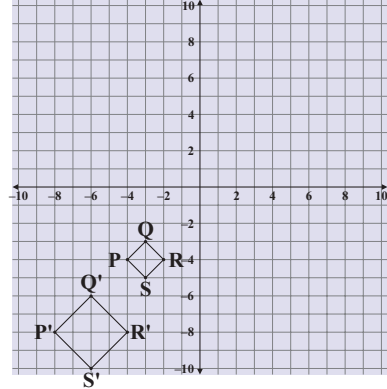
Remember, letters without primes (ABC, XYZ) are an "old" image, before transformation. Letters with primes (A'B'C', X'Y'Z') are a "new" image after transformation.

1. For each diagram, determine the scale factor for scaling the old image to the new image.

a.

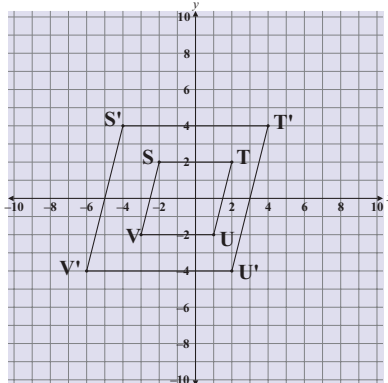


b.

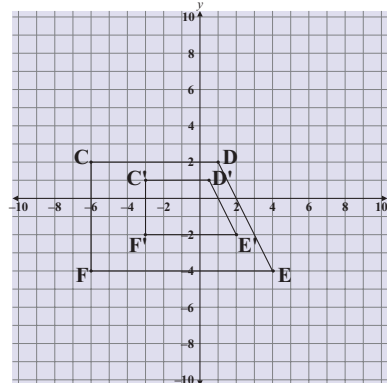


You can compare the length of a side or a diagonal on the new image to the length of a corresponding side on the old image.

c.



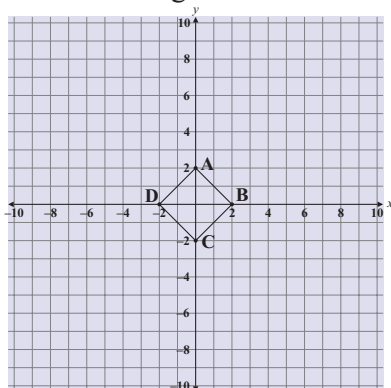
d.



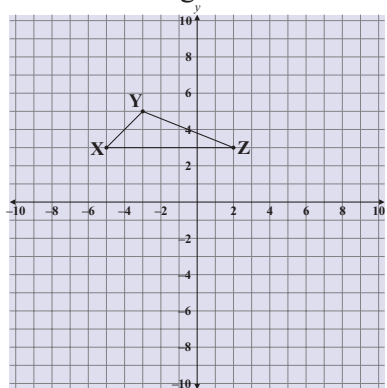
2. What is the difference between applying a scale factor and applying one of the other three transformations (reflection, translation, and rotation)?

3. Jeanette printed out a schedule for the household chores. The shedule measured 8 cm by 8 cm. Jeanette's roommates complained that the shedule was not big enough to read easily, so Jeanette enlarged it by a scale factor of 2.5. Find the dimensions of the enlarged shedule.

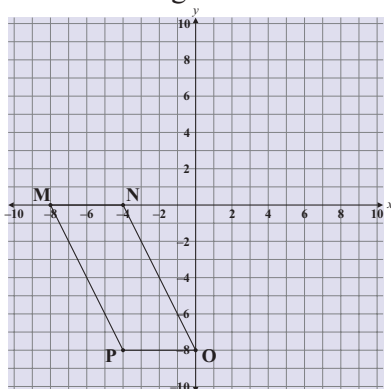
4. a. Scale square ABCD by a factor of 5.
Label the image A'B'C'D'.



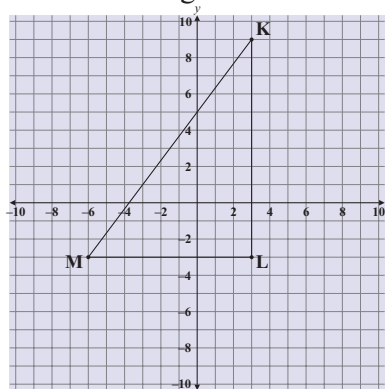
- b. Scale triangle XYZ by a factor of 2.
Label the image X'Y'Z'.



- c. Scale parallelogram MNOP by a factor of $\frac{1}{4}$.
Label the image M'N'O'P'.



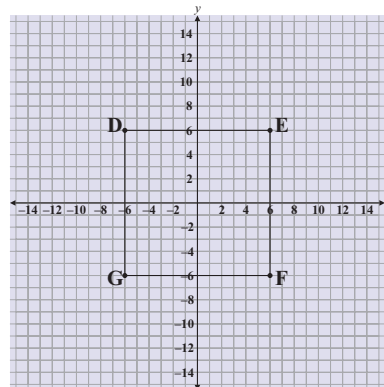
- d. Scale triangle KLM by a factor of $\frac{2}{3}$.
Label the image K'L'M'.



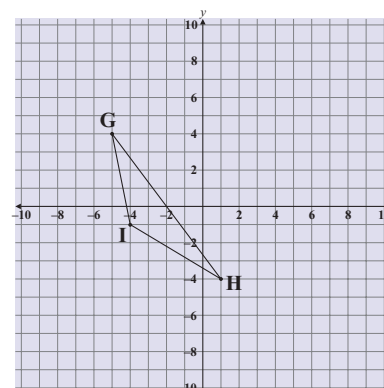
5. Complete the following transformations then fill in the table describing the images.

- a. Scale DEFG by a factor of 2. Label the image D'E'F'G'.
b. Scale DEFG by a factor of 0.5. Label the image D''E''F''G''.
c. Scale D'E'F'G' by a factor of 0.5. Label the image D'''E'''F'''G'''.
d.

	DEFG	D'E'F'G'	D''E''F''G''	D'''E'''F'''G'''
Side length	12 units			
Perimeter				
Area				



6. Transform triangle GHI according to: $(x, y) \rightarrow (2x + 3, -y)$.
Label the image G'H'I'.



Lesson
3.4.4

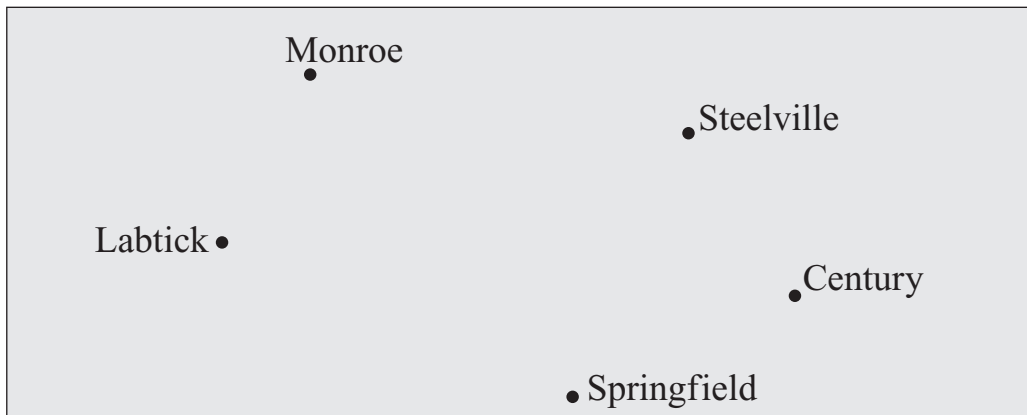
Scale Drawings

California Standards: Measurement and Geometry 1.2

- Mr. Horner is remodeling his kitchen. One wall will have floor cabinets measuring 3 feet by 8 feet. Mr. Horner wants to make a scale drawing of the cabinets using a scale of 1 inch : 4 feet.
 - What will the dimensions of the cabinets be in the scale drawing?
 - Make the scale drawing of the floor cabinets using a scale of 1 inch : 4 feet.

Use a ruler, be accurate, and label your diagram with side lengths.

- Five cities are shown on the map below using a scale of 1 inch : 25 miles.



- What is the distance from Steelville to Monroe?
 - How far would you travel if you went in a straight line from Monroe to Labtick and then in a straight line from Labtick to Century?
.....
 - How much farther is Springfield to Steelville than Steelville to Century?
 - On a map with a scale of 1 inch : 40 miles, would the cities be closer or farther apart than they are on the map shown? Explain your answer.
.....
- Taylor draws a 30 ft by 25 ft rectangle using a scale of 3 in. : 15 ft. What dimensions will the rectangle in her scale drawing have?
.....
 - Redraw a square with side length of 200 cm using a scale of 4 mm : 32 cm.

Example

Adrienne has a social studies assignment to make a map of the classroom. The room is 33 feet long. What scale could she use so that the room is 11 inches long on the map?

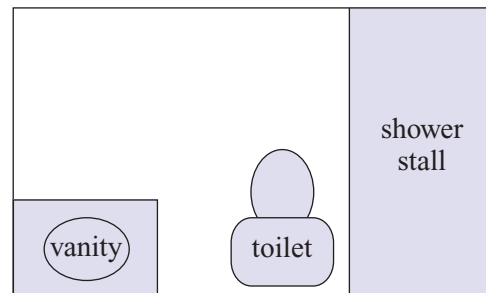
Solution

Set up and solve a proportion:

$$\begin{aligned}\frac{\text{Drawing length}}{\text{Real-life length}} &= \frac{11 \text{ in.}}{33 \text{ ft}} = \frac{1 \text{ in.}}{x} \\ 11 \text{ in.} \times x &= 1 \text{ in.} \times 33 \text{ ft} \\ x &= \frac{1 \text{ in.} \times 33 \text{ ft}}{11 \text{ in.}} \\ x &= 3 \text{ ft}\end{aligned}$$

A scale of 1 inch : 3 feet would be appropriate.

4. Mrs. Felsher plans to remodel her bathroom. The diagram shows the current bathroom drawn to a scale of $\frac{1}{4}$ in : 1 ft.
- a. What are the dimensions of the current shower stall in feet?



- b. Mrs. Felsher would like to replace her old vanity with a vanity two feet longer. If Mrs. Felsher does not move her toilet, will the new vanity fit in the same place in her bathroom? Explain.

- c. What is the area of the bathroom in feet?

5. When making a scale drawing of a building measuring 20 ft by 40 ft using a scale of 1 in : 10 ft, Harry made the following calculations for the dimensions. Is Harry's working correct? Explain your answer.

$$\begin{aligned}\text{Width: } \frac{1}{10} &= \frac{x}{20} \\ x &= 2 \text{ in.}\end{aligned}$$

$$\begin{aligned}\text{Length: } \frac{1}{10} &= \frac{y}{40} \\ y &= 4 \text{ in.}\end{aligned}$$

6. A scale drawing of a rectangular backyard measures 9 inches by 12 inches. The scale for the drawing is 1 inch : 3 feet. What are the dimensions of the real backyard?
7. Paulo and Jeremy are comparing different routes between their houses. Paulo finds a route measuring 3.7 inches using a map with a scale of 1 inch : 25 miles. Jeremy finds a different route measuring 17 cm on a map with a scale of 1 cm : 5 miles. Whose route is shorter? By how far?

Lesson
3.4.5

Perimeter, Area, and Scale

California Standards: Measurement and Geometry 1.2, 2.0

Example

Roger draws an image of square A using a scale factor of 3. Find the perimeters of square A and the image A'.

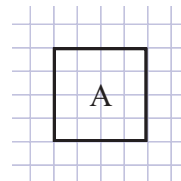
Solution

The sides of square A measure 4 units.

Perimeter of A: $4 \times 4 = 16$ units

The sides of square A' measure $4 \times 3 = 12$ units.

Perimeter of A': $4 \times 12 = 48$ units

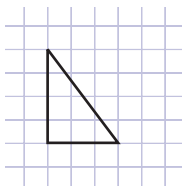


1. Find the perimeter of the following images after the scale factor given has been applied.

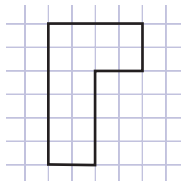
a. Scale factor 2

b. Scale factor 0.25

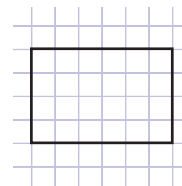
c. Scale factor 3.5



.....



.....



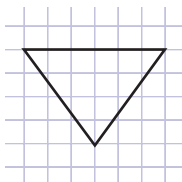
.....

2. Find the area of the following images after the scale factor given has been applied.

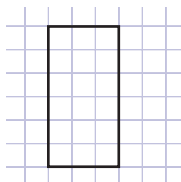
a. Scale factor 2

b. Scale factor 5

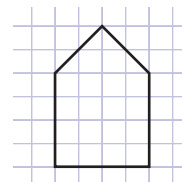
c. Scale factor 25



.....



.....



.....

3. Michelle draws a figure with a perimeter of 6 cm. Find the perimeter of the image if Michelle scales her figure by the following factors.

a. Scale factor 0.5

b. Scale factor 12

c. Scale factor 3

4. Lakesha draws a figure with an area of 11 cm^2 . Find the area of the image if Lakesha scales her figure by the following factors.

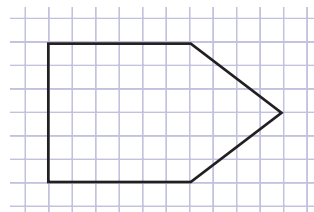
a. Scale factor 2

b. Scale factor 0.25

c. Scale factor 10

5. The diagram shows a scale drawing of an object. Find the perimeter of the actual object using the scale 1 unit : 3 inches.

.....



6. Keri makes a scale drawing of a room with a perimeter of 28 yards and an area of 144 square yards, using a scale of 4 inches : 1 yard.

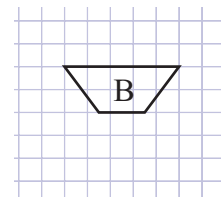
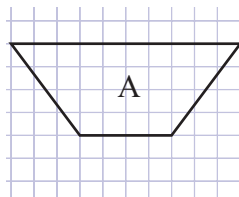
a. What is the perimeter of the room in Keri's drawing?

b. What is the area of the room in Keri's drawing?

7. The diagram shows two similar figures. The perimeter of figure A is 48 cm and the area of figure B is 84 cm^2 .

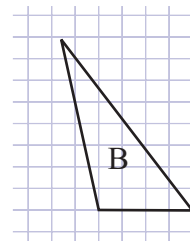
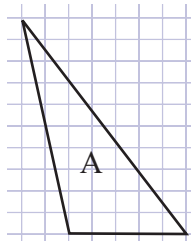
a. What is the perimeter of figure B?

b. What is the area of figure A?



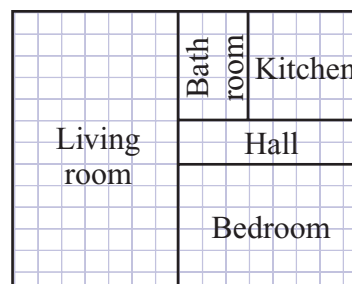
8. The diagram shows two similar triangles. The area of triangle A is 50 ft^2 . What is the area of triangle B?

.....



9. The diagram shows a scale drawing of an apartment with a scale of 1 unit : 2 feet. What are the real dimensions of the bedroom?

.....



Lesson
3.4.6

Congruence and Similarity

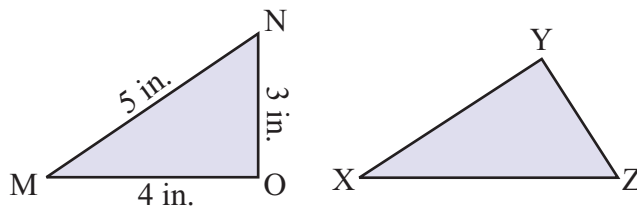
California Standards: Measurement and Geometry 3.4

Example

The triangles in the diagram are congruent. Determine the length of side XZ.

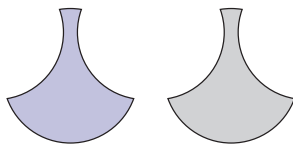
Solution

The two triangles are congruent, so all corresponding sides must be the same length. XZ is the longest side of triangle XYZ, so must be equal in length to the longest side of MNO. $XZ = MN = 5$ in.

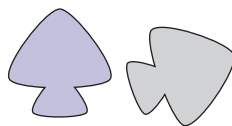


1. State whether each pair of figures is congruent.

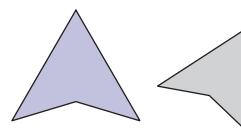
a.



b.

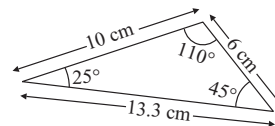
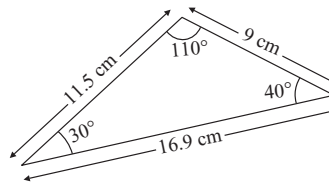


c.



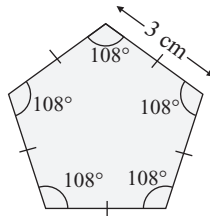
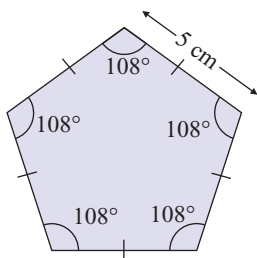
2. Why are these triangles not similar?

.....
.....



3. Identify the shapes as congruent, similar, or neither. Give a reason for each answer.

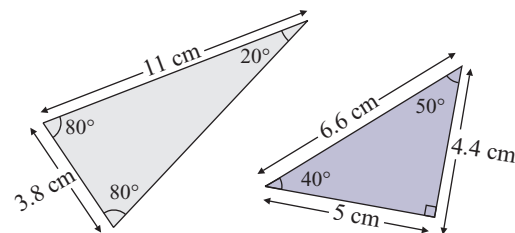
a.



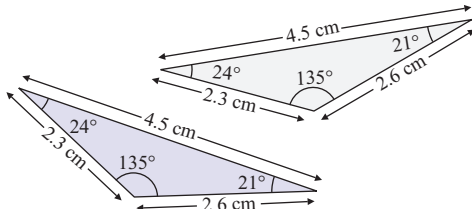
.....
.....

b.

.....
.....



c.

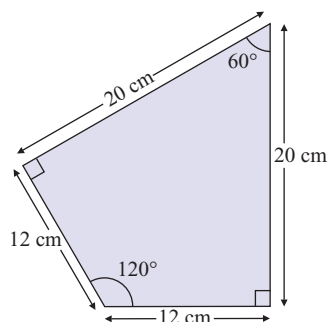


.....
.....

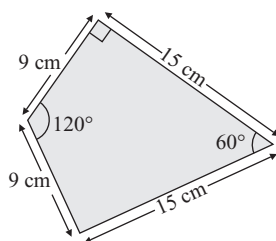
4. Explain the difference between similar figures and congruent figures.

5. Which two shapes below are congruent? Explain why the others aren't.

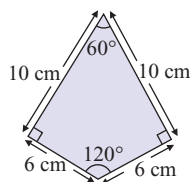
A.



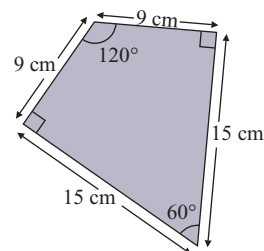
B.



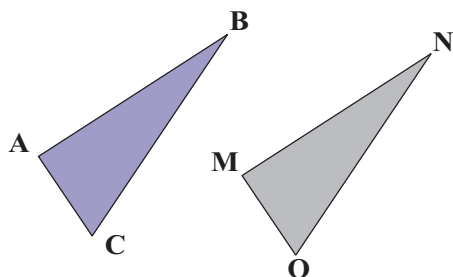
C.



D.

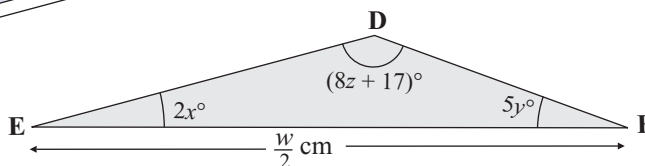
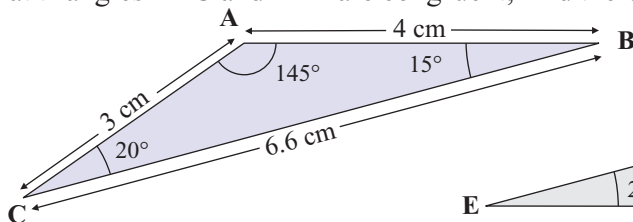


6. Triangles ABC and MNO are congruent. State the pairs of angles and sides that are identical.



7. Is every pair of regular hexagons similar, congruent, or neither? Explain.

8. Given that triangles ABC and DEF are congruent, find the values of w , x , y , and z .



Lesson
3.5.1

Constructing Circles

California Standards: Measurement and Geometry 3.1

1. Use a compass to construct circles with the given parameters.

- a. Radius of 2 centimeters. b. Diameter of $1\frac{1}{2}$ inches. c. Radius of $\frac{5}{8}$ inches.

2. Concentric circles are circles with the same center but different length radii.

- a. Construct concentric circles with radii of 1.5 and 3 centimeters.

Label the smaller circle K and the larger circle L.

- b. Find the circumference and area of each circle.

Express your answers in terms of π .

Circumference of K:

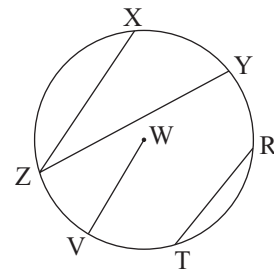
Area of K:

Circumference of L:

Area of L:

3. Identify all the chords in the circle in the diagram.

.....
.....



4. a. Construct a circle, N, with a radius of 15 mm.

- b. Add a chord LT in circle N with a length of 25 mm.

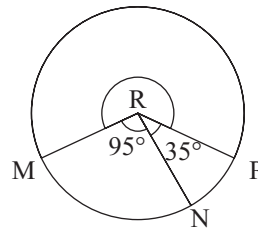
Example

In circle R find the measure of the large angle MRP.

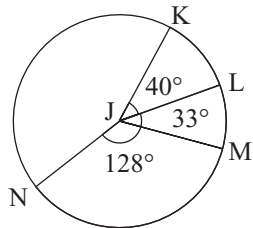
Solution

$$360 - (95 + 35) = 230^\circ \quad \text{Subtract the given angles from } 360^\circ.$$

The sum of all the central angles in a circle is 360° .

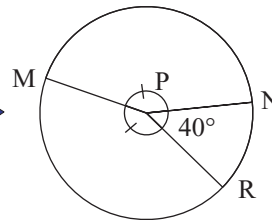


5. a. Find the measure of angle NJK.

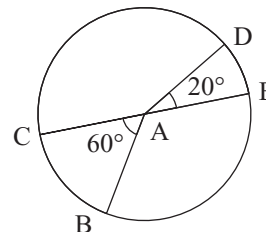


These diagrams are not drawn perfectly to scale. If you try to cheat by using a protractor, you will get the answer wrong.

- b. Find the measure of angle MPN.

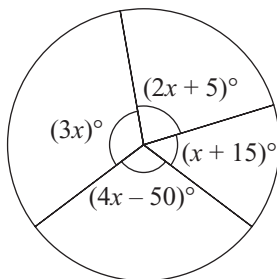


- c. CE is the diameter of circle A. Find the measure of angle EAB and the measure of angle CAD.



A diameter cuts a circle in half.

6. Use the angles given in the circle to find the value of x .

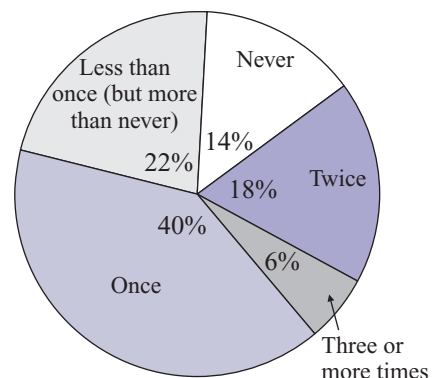


7. The circle graph shows the results of a survey of 500 people. The survey asked on average how many nights a week they eat at a restaurant.

- a. How many people surveyed said they eat at a restaurant on average once a week?

- b. How many people said they eat at a restaurant on average two or more times a week?

- c. What is the angle of the sector labeled "Never"?



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Lesson

3.5.2 Constructing Perpendicular Bisectors

California Standards: Measurement and Geometry 3.1

1. Use a compass and straightedge to copy the line AB. Label your copy CD.

A—————B

2. Determine whether the following statements are true or false. Explain any false statements.
- a. A midpoint divides a line into two equal parts.

.....

b. A line segment has a beginning and an end.

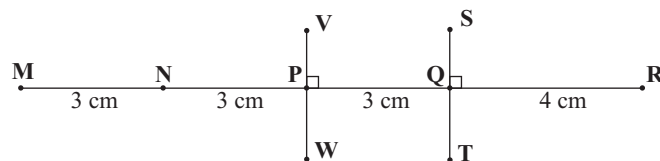
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c. A perpendicular bisector passes through a line segment at its midpoint and forms a right angle.

.....

d. A line segment contains a finite number of points.

3. The following questions refer to this diagram:

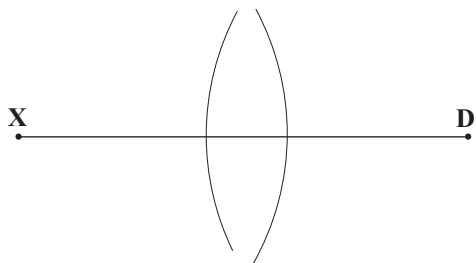


- a. Identify the midpoint of line segment MP.
- b. Identify the midpoint of line segment NQ.
- c. Identify a line segment perpendicular to line segment ST.
- d. Identify the perpendicular bisector of line segment NQ.
- e. Is P the midpoint of line segment MR? Explain your answer.

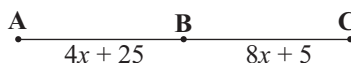
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f. Why is line segment VW a perpendicular bisector of NQ but not MR?

4. Roberta is trying to construct a midpoint for the line segment XD. What mistake has Roberta made?



5. In this diagram, B is the midpoint of AC.



Find the value of x

6. a. Find the midpoint of segment FG using a straightedge and compass. Label the midpoint H.



- b. Construct a perpendicular bisector of line segment JK.
Label the perpendicular bisector PY.
Give the point of intersection of JK and PY the label U.



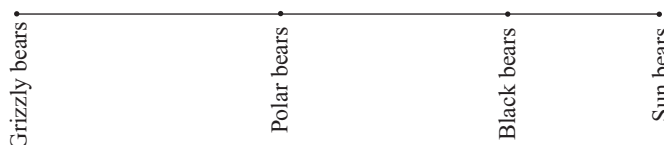
7. a. Construct a segment of length 6 centimeters.
Label the endpoints R and W.

- b. Construct a perpendicular bisector of RW.
Label the perpendicular bisector PY.
Give the point of intersection of RW and PY the label U.

8. The diagram shows a scale drawing of a line segment MN. S is the midpoint of MN, V is the midpoint of SN, and Y is the midpoint of SV. If MN is 48 inches long, find the length of YV.



9. A zoo has recently expanded and wants to place a bench at the midpoint between the new grizzly bear and polar bear enclosures, and another bench at the midpoint of the new black bear and sun bear enclosures. Use constructions to show where the benches should be placed on the diagram.



Lesson
3.5.3

Perpendiculars, Altitudes and Angle Bisectors

California Standards: Measurement and Geometry 3.1

1. a. Construct a line segment perpendicular to QR that passes through the point W. Label the perpendicular segment ST.



After your initial two marks on segment QR, increase the radius of the compass to find the intersecting arcs about point W.

- b. Construct a line segment perpendicular to AC that passes through the point E. Label the perpendicular segment FG.

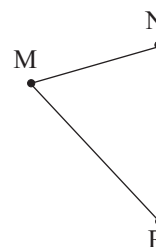


2. A construction company is developing a new subdivision on Main Street. The two streets that will intersect Main Street are marked with crosses. Construct line segments perpendicular to Main Street at the labeled points to show the two new streets. Label the streets Apple Street and Cherry Street.

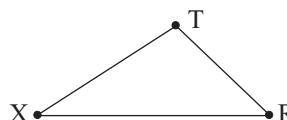


3. Construct and label the following altitudes.
a. Altitude MZ from vertex M to segment PN.

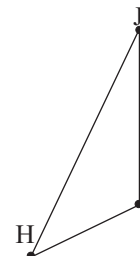
It will help if you extend side PN with a dashed line.



- b. Altitude TY from vertex T to segment RX.



- c. Altitude HK from H to the line containing segment IJ.



4. In what type of triangle does an altitude of the triangle intersect a side of the triangle at its midpoint? Explain your answer.

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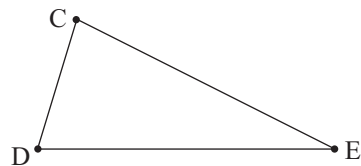
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5. a. Explain why it is useful to extend segment DE to construct an altitude from C in triangle CDE?

.....

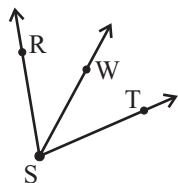
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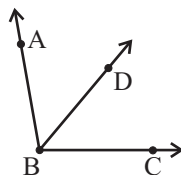
- b. Construct an altitude from vertex C in triangle CDE.

6. a. Ray SW bisects angle RST and the measure of angle RST is 76° . Find the measure of angle RSW.



.....

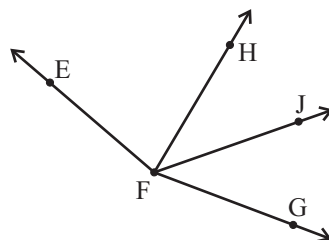
- b. Ray BD bisects angle ABC and the measure of angle ABC is 100° . Find the measure of angle ABD.



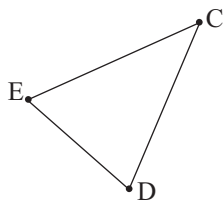
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- c. Ray FH bisects angle EFG and ray FJ bisects angle HFG. The measure of angle EFG is 160° . Find the measure of angle JFG.

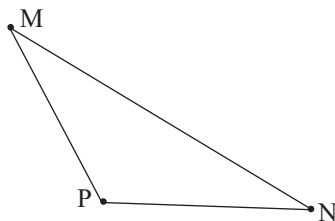
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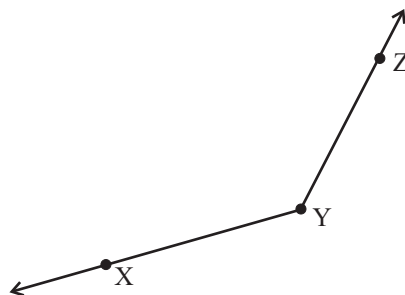
7. a. Construct an angle bisector for angle DCE in triangle CDE.



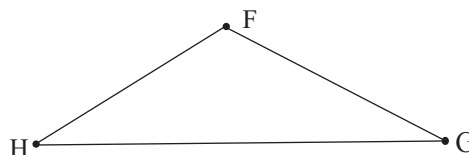
- b. Construct an angle bisector for angle PMN in triangle MNP.



8. a. Bisect angle XYZ. Label the bisecting ray YW.
b. Bisect angle WYZ. Label the bisecting ray YT.
c. Bisect angle TYZ. Label the bisecting ray YR.



9. Construct an angle bisector at vertex H in triangle FGH.



Lesson

3.6.1

Geometrical Patterns and Conjectures

California Standards: Mathematical Reasoning 1.2, 2.4, Measurement and Geometry 3.3

Example

State a general conjecture about the number sequence that begins 7, 9, 11, 13, then find the next term.



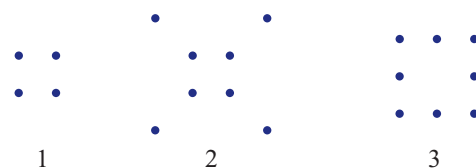
A general conjecture describes something to do with the overall pattern using a sentence or a formula.

Solution

The pattern can be described in mathematical notation as $2n + 5$.

The next term is $2 \times 5 + 5 = 15$.

1. The diagram shows the first three instances of a pattern. Draw two different possible fourth instances and justify each choice.



.....

.....

.....

.....

.....

.....

2. Given the sequence beginning 7, 12, 17, 22, 27, state one specific conjecture about this sequence.

.....

3. A pattern is formed according to the formula $n^2 - 3$.
If $n = 0, 1, 2, 3, \dots$ find the first 5 terms in this pattern.

.....

4. The table below gives the hours of sunlight five tomato plants received per day and the number of tomatoes they produced. State one general and one specific conjecture about the data.

Plant	A	B	C	D	E
Hours of Sunlight	3	4	5	6	7
Tomatoes produced	17	23	35	42	50

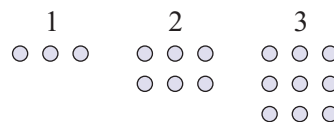
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5. Test the conjecture that all whole numbers have an even number of factors.

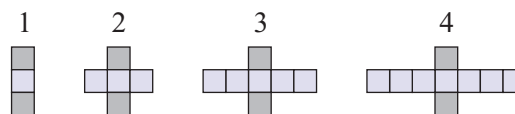
6. The diagram shows the first three instances of a dot pattern.



- a. State two general conjectures about the pattern.

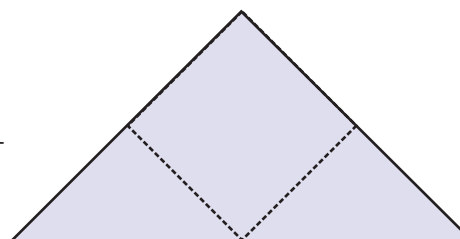
- b. Draw the fourth instance of the pattern.

7. Give a general conjecture about the pattern in the diagram using mathematical notation.



8. A number sequence begins 1, 8, 27, 64, 125.
Give the next two terms in the sequence and justify your answers.

9. Prove the following conjecture: “The area of an isosceles right triangle is twice the area of the square it contains.”



10. Test the following conjecture about perfect squares:
“Given three consecutive perfect squares a , b , and c , the sum of a and b is never equal to c ”.

Lesson
3.6.2

Expressions and Generalizations

California Standards: Algebra and Functions 1.1, Mathematical Reasoning 2.2, 3.3

Example

Use the following pattern to answer the questions.



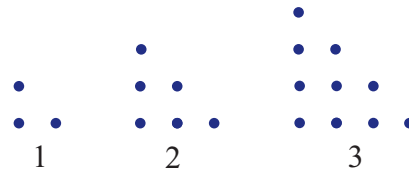
- Give two specific conjectures for this pattern.
- Give two general conjectures for this pattern.

A specific conjecture might be a prediction of a future instance.

A general conjecture describes the overall pattern using a sentence or a formula.

Solution

- The fifth instance is a polygon with seven sides.
The sixth instance is an octagon.
- Each instance is a polygon that has one more side than the previous polygon.
The formula for the number of sides is $n + 2$ where $n = 1, 2, 3, \dots$



- The diagram shows the first three instances of a dot pattern.

- Give two specific conjectures for this pattern.

- Give two general conjectures for this pattern.

- The first four terms of a sequence are 5, 8, 11, 14.

- Give two specific conjectures for this pattern.

- Give two general conjectures for this pattern.


- Write a formula to describe this pattern.


- Given the sequence beginning 45, 40, 35, 30:


- Write an expression to describe the sequence.


- Use the expression to find the tenth term.

4. A pattern is formed by the expression $5n + 2$, where $n = 1, 2, 3, \dots$
- Find the first three numbers in the pattern.
 - Using a sentence, describe how to find the fourth term.
.....
 - Write a specific conjecture for this pattern.
.....
 - Write a general conjecture for this pattern.
.....

5. A pattern of blocks is shown on the right.
- 
1





2



3



4
- Describe the pattern of blocks using words.
.....
 - Use a mathematical expression to describe the pattern.


6. Write and evaluate a formula to find the 5th term of the pattern $1, \frac{5}{3}, \frac{7}{3}, 3, \dots$

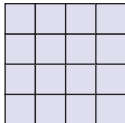
7. The pattern of dots shown increases by two for each new instance.
- Make a table showing n ,
 $n + 2$ and number of dots.


1
 
2
 
3
- Why does the formula $n + 2$, $n = 1, 2, 3$ not work for this pattern?
.....
 - Write a mathematical expression to describe the pattern.

8. The diagram shows a pattern of blocks.
- Write an expression to describe the number of blocks in each instance of the pattern.
.....
- 
1


2


3


4
- Draw the sixth instance in the pattern.
 - Find the number of blocks in the 12th instance in this pattern.

9. A pattern is formed by the formula $\frac{4n-5}{2}$. If $n = 0, 1, 2, 3, \dots$ find the first five terms in this pattern.
.....

Lesson 4.1.1

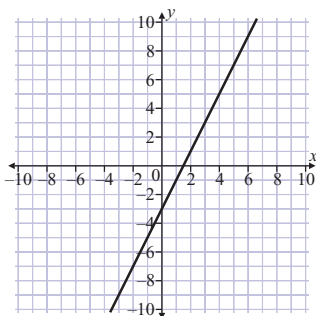
Graphing Equations

California Standards: Algebra and Functions 1.5, 3.3

1. The equation $y = mx + b$ consists of variables and constants.
Write down which is which, and state what a graph drawn using this equation would look like.

Variables: Constants: Graph Type:

2. Use the following graph to answer the questions in this Exercise.



Do these ordered pairs satisfy the equation whose line is shown on the graph?

a. (4, 5)

b. (1, 2)

c. (0, 3)

d. (-2, -7)

.....

- e. Does the graph show the equation $y = 2x - 3$?

.....

3. State whether the following equations are linear equations or not:

a. $y = x - 2$

b. $y = 5x^2 + 6$

c. $y = 50 - \frac{x}{2}$

d. $3 + 7x = y^2$

e. $4 = y - x$

f. $xy = 3x^2, x \neq 0$

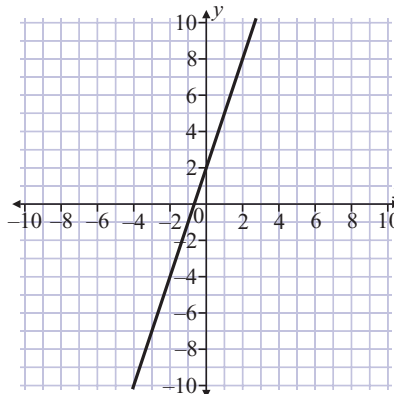
Example

Graph the equation $y = 3x + 2$.

Solution

Make an x - y table with five x -values, substitute each x -value into the equation to find the y -values, then plot (x, y) — the ordered pairs.

x	y	(x, y)	Work
-2	-4	$(-2, -4)$	$y = 3(-2) + 2 = -6 + 2 = -4$
-1	-1	$(-1, -1)$	$y = 3(-1) + 2 = -3 + 2 = -1$
0	2	$(0, 2)$	$y = 3(0) + 2 = 0 + 2 = 2$
1	5	$(1, 5)$	$y = 3(1) + 2 = 3 + 2 = 5$
2	8	$(2, 8)$	$y = 3(2) + 2 = 6 + 2 = 8$



4. a. Find solutions to the equation $y = -2x + 1$ using the x -values in the table.

x	y	Ordered Pair
-4		
-2		
0		
2		
4		

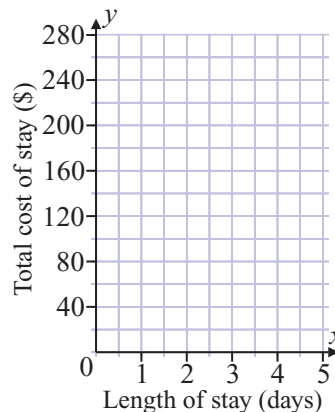
- b. Use these to write ordered pairs that could be used to plot the graph of this equation.

5. The cost of housing a dog at the Pet Motel while on vacation is \$42 a day and is represented by the equation $y = 42x$, where x represents the number of days.

x	y	Ordered Pair
0		
1		
2		
3		
4		
5		

- a. Complete the table on the left for this equation.

- b. Graph the points in the table on the given axes.

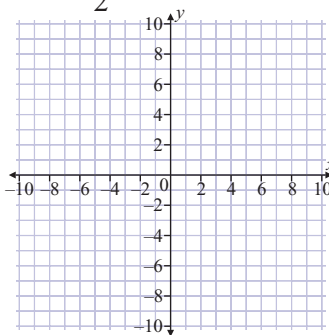
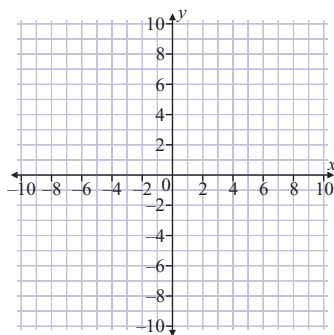
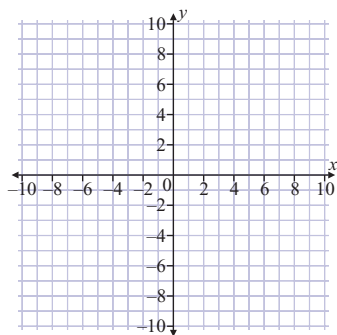


6. Graph each equation on the axes provided:

a. $y = 2x - 1$

b. $y = x + 3$

c. $y = \frac{1}{2}x - 1$



- d. Complete the sentence:

As the x -value in each graph increases, each y -value

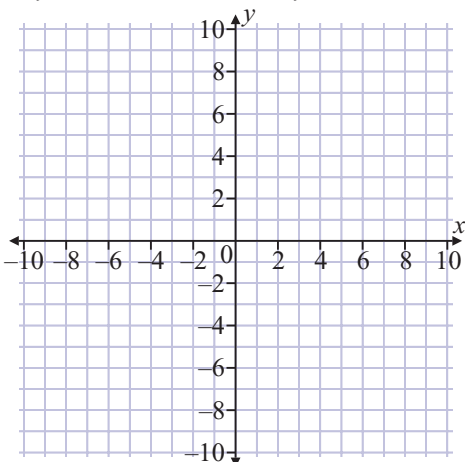
7. Graph and label each line on the axes below:

a. $x = 4$

b. $x = -3$

c. $y = 1$

d. $y = -5$

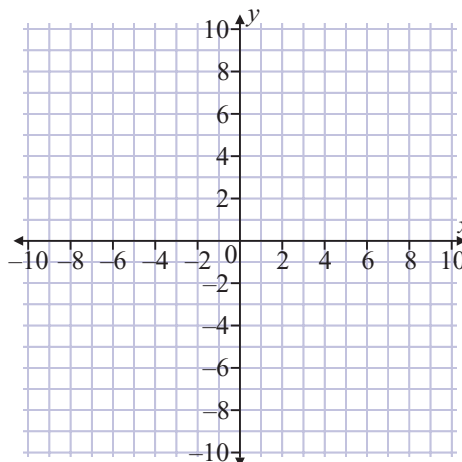


8. Graph and label each line on the axes below:

a. $y = -x$

b. $y = -4x - 3$

c. $y = -\frac{1}{2}x + 1$



Lesson
4.1.2

Systems of Linear Equations

California Standards: Algebra and Functions 1.1, 1.5

Example

Write a system of linear equations to represent the following statement:

The sum of x and y is 6, and y is twice x .

Solution

You need to write two equations, both of which must be true for the statement to be true.

The first part says, “the sum of x and y is 6,” so $x + y = 6$.

The second part says, “ y is twice x ,” so $y = 2x$.

These two equations form a system of linear equations.

1. Say whether each statement is true or false. In a system of two linear equations:

a. There is exactly one solution.

b. Parallel lines can have a solution.

c. The solution is the point of intersection of the graphs of the equations.

d. All points on each of the line graphs have x and y values that are a solution to the system.

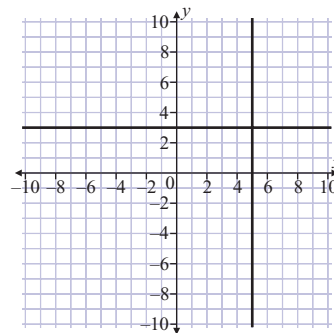
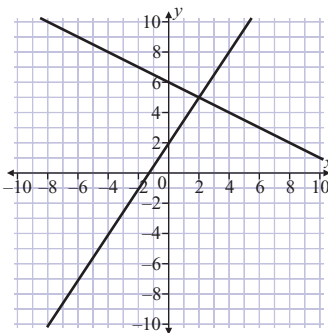
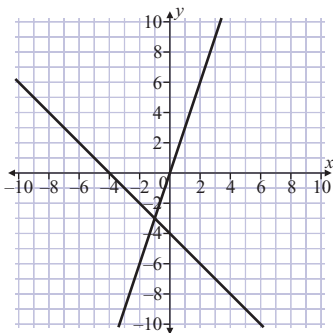
2. Write a system of equations to represent the following situation: Melanie buys 2 CDs at \$ x each and 3 posters at \$ y each. She is charged \$45 altogether. A CD costs \$3 more than a poster.

3. Use the graphs to identify the solutions of the systems of equations below. Substitute your solutions into the equations to check.

a. $y = 3x$ and $y = -x - 4$

b. $y = \frac{3}{2}x + 2$ and $y = -\frac{1}{2}x + 6$

c. $y = 3$ and $x = 5$



4. Determine whether each point is a solution to the given system of equations.

a. $(0, 3)$ $y = 5x + 3$ $y = -x + 3$

b. $(-5, -4)$ $y = \frac{2}{5}x - 2$ $y = -2x + 6$

c. $(2, 2)$ $y = \frac{1}{2}x + 1$ $y = -2x + 6$

5. $(-3, 4)$ is the solution to the following systems of equations: True or false?

a. $y = \frac{1}{3}x + 5$ and $y = -\frac{4}{3}x$

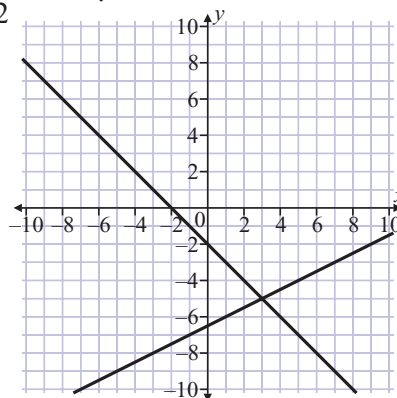
b. $y = x + 7$ and $y = -\frac{1}{2}x + 7$

c. $y = -x + 1$ and $y = -\frac{1}{3}x + 3$

6. This question is about the system of equations $y = -x - 2$ and $y = \frac{1}{2}x - 6.5$.

a. Use the graph to find the solution to the system of equations.

b. Use algebra to verify the solution to the system of equations.



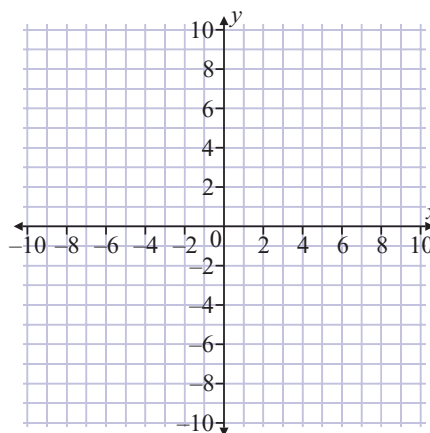
7. If $(-2, -3)$ is a solution to the system of equations $y = \frac{3}{2}x$ and $y = -4x - 11$ then explain the point's relationship graphically to the system of equations.

8. Solve the following system of equations by graphing.

$$y = -\frac{2}{3}x - 3 \quad \text{and} \quad y = -2x + 5$$

x	y	Ordered Pair
-6		
-3		
0		
3		
6		

x	y	Ordered Pair
-2		
-1		
0		
1		
2		



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The solution is

Lesson
4.1.3

Slope

California Standards: Algebra and Functions 3.3

1. Identify each line as having positive, negative, zero, or undefined slope.

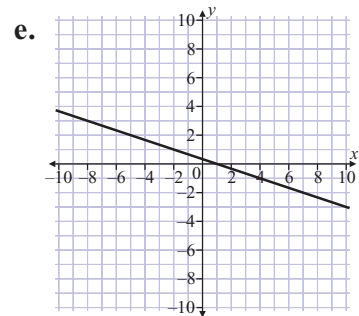
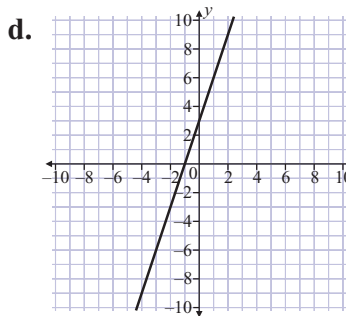
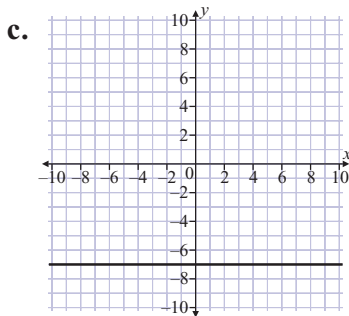
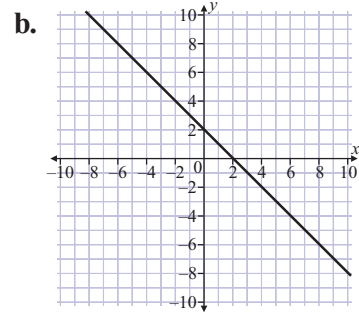
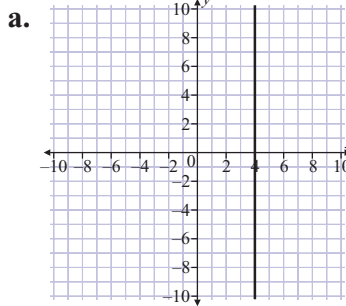
a.

b.

c.

d.

e.



Example

Find the slope of the straight line joining the points (3, 7) and (-2, 5).

Solution

Label (3, 7) point 1. $x_1 = 3$ and $y_1 = 7$.

Label (-2, 5) point 2. $x_2 = -2$ and $y_2 = 5$.

$$\frac{(5-7)}{(-2-3)} = \frac{-2}{-5} = \frac{2}{5}$$

The slope formula is $\frac{(y_2 - y_1)}{(x_2 - x_1)}$.

2. Graph and label the following on the axes to the right:

a. a line with zero slope

b. a line with undefined slope

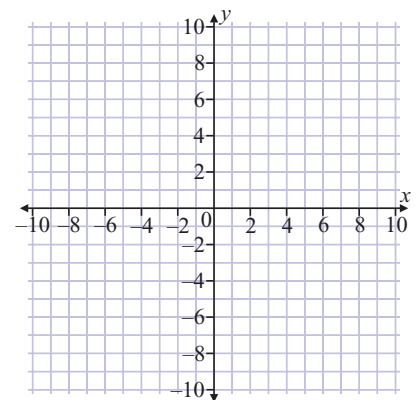
c. a line with a slope of $\frac{2}{5}$

3. a. A line with slope $\frac{1}{2}$ contains the point (7, 2).
Find two other points on this line.

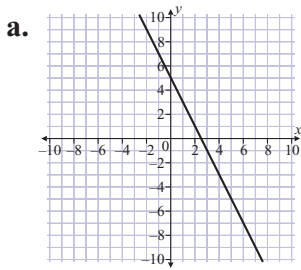
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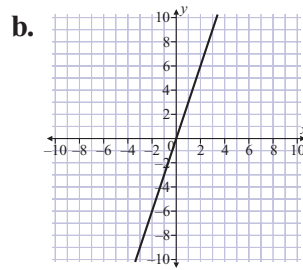
b. A line with slope $-\frac{1}{4}$ contains the point (-2, 4).
Find two other points on this line.

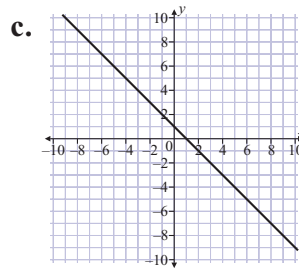
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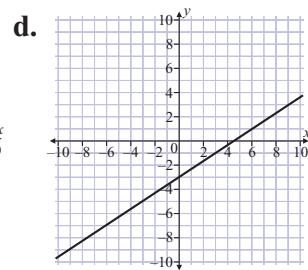


4. Find the slope of each line using any two points on the line.
Check your answer using the slope formula.









5. This question is about the straight line joining the points $(8, -3)$ and $(-7, -6)$.

- a. Find the slope using $(8, -3)$ as point 1 in the slope formula.

- b. Find the slope using $(-7, -6)$ as point 1 in the slope formula.

- c. Why did the slope between the two points remain the same when point 1 was redefined?

6. Find the slope between each pair of points. Express each slope in its simplest form.

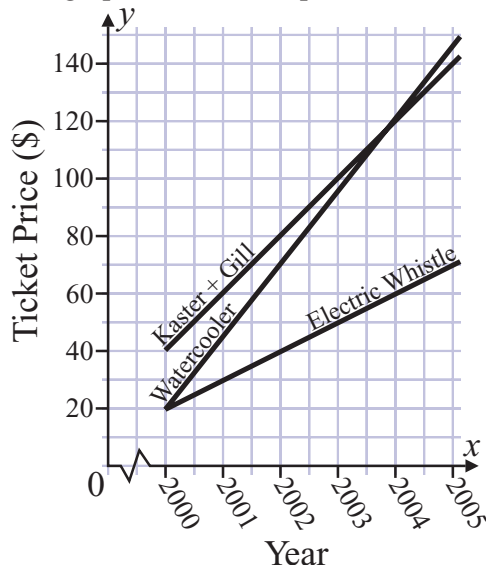
- a. $(0, 6)$ and $(7, 9)$

- b. $(-3, -2)$ and $(4, 12)$

- c. $(4, 4)$ and $(3, 9)$

- d. $(12, -1)$ and $(-13, -11)$

7. The graph shows ticket prices for three bands from 2000 to 2005.



- a. Which band's ticket price increased the least amount?
By how much did it increase?

- b. Which two bands had the same ticket price in 2000?

- c. How much does Watercooler's ticket price increase by each year?

- d. If the price increase continues at the same rate then what will be the price of a ticket to see the band Electric Whistle in 2008?

Lesson
4.2.1

Ratios and Rates

California Standards: Measurement and Geometry 1.3

1. The ratio of pencils to pens in a pencil case is 5 to 2. Express this ratio in two other ways.
.....
2. The ratio of male to female actors in a theatrical production is 4 : 7. What is the ratio of female to male actors in the same production?
.....

Example

Find the unit rate equivalent to 4 boxes of cereal for \$10.

Solution

$$\frac{\$10}{4 \text{ boxes}} = \frac{\$10}{4} / \text{box} = \$2.5 / \text{box}$$

So the rate is **\$2.50 per box**.

Note that 0.4 boxes per dollar would also be a valid answer to this question.

3. A car travels 150 miles in 4 hours. Express this in miles per hour.
.....
4. A bicycle costs \$35 to rent for 4 hours. Express this as a unit rate.
.....
5. Find the unit rate in the following situations:
 - a. \$5.00 for 10 yogurt cups.
.....
 - b. \$21.42 for 9 frozen dinners.
.....
 - c. Frank runs 5 miles in 40 minutes.
.....
 - d. Nicole reads 20 pages in 80 minutes.
.....
6. A camera store prints digital pictures for \$0.30 each or \$38 for 200 pictures. How much will a customer save per print by paying \$38 for 200 prints, rather than buying 200 prints at \$0.30 per print?
.....

7. Grocery store A has 5-pound turkeys for \$10.95 and Grocery store B has 7-pound turkeys for \$15.96. Which grocery store is offering the better rate per pound?

8. Allen has to choose between two gym membership plans. Determine which of these two plans is the better deal: 20 visits for \$50 or 25 visits for \$60.

9. A pizza restaurant offers the following two deals:

Deal #1

\$8.99 for one
large pizza

Each additional
large pizza on the
same order – \$5 each

Deal #2

Fixed price of
\$6.00 per large pizza

- a. At what number of pizzas does Deal #1 become a better deal?

*It may help to make tables
to compare the two deals.*

- b. What is the cost per pizza if ordering 10 pizzas using Deal #1?

10. Find the unit rate:

- a. 5 dozen eggs cost \$10.60.

- b. Nine months of cell phone service costs \$358.65.

- c. A package of 8 bread buns costs \$1.36.

11. The science museum has a yearly attendance of 350,000 people. The zoo has a yearly attendance of 390,000. The science museum is closed nine days a year and the zoo is open every day of the year. Which attraction has the higher average daily attendance for the days that it is open?

Lesson
4.2.2

Graphing Ratios and Rates

California Standards: Algebra and Functions 3.4

Example

Luann is a landscaper and charges \$40 per hour.

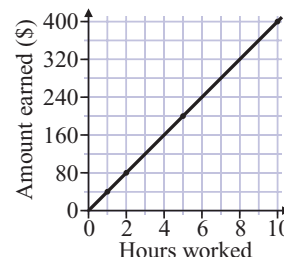
Plot a graph to show how the amount Luann earns increases with the amount of time she works.

Solution

Step 1: You know she earns \$40 per hour so make a table of her earnings for a varying number of hours.

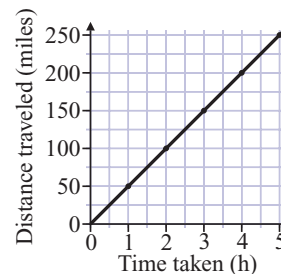
# hours	Amount earned
1	\$40
2	$2 \times 40 = \$80$
5	$5 \times 40 = \$200$
10	$10 \times 40 = \$400$

Step 2: Plot a graph from your table to show the number of hours worked against the amount earned.



- Luann worked 8 hours on Friday.
Use the graph from the Example above to determine how much she earns.

- The graph shows the progress of a carpool van that is traveling at a constant rate. Use the graph to find how many miles per hour the van is traveling.

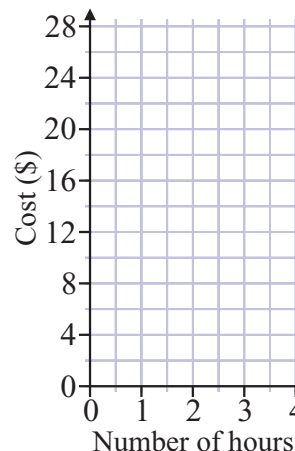


- A babysitter charges \$5.00 per hour for one child and \$7.00 per hour for two children.
 - Complete the tables.

- Graph both tables on the same axes.

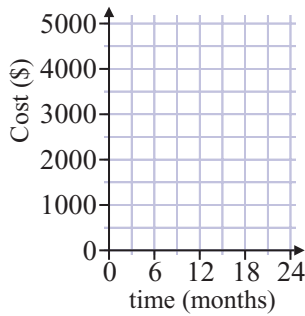
# hours	One Child
0	
1	
2	
3	
4	

# hours	Two Children
0	
1	
2	
3	
4	



- Why is the line for babysitting two children steeper than the line for babysitting one child?

4. The average cost of a certain computer has decreased since it was first on sale.

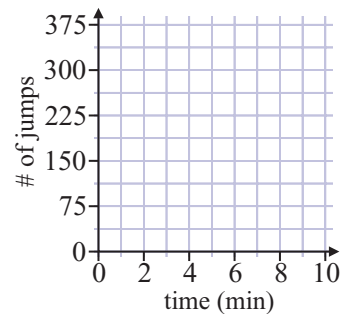


The following table gives the sale value in the months since it was first released. Use the table values to plot the points on a graph. Draw a line through the points and find the slope of the line.

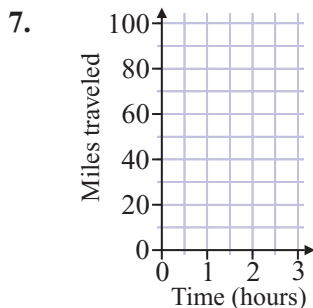
months	cost (\$)
6	4500
12	3500
18	2500
24	1500

5. Marcus has started a new fitness program. He decided to jump rope every morning and record his progress in the table given. If Marcus jumps at the same rate throughout each session, find the missing values to complete the table and graph it.

Time (mins)	# of jumps
2	75
	150
6	
8	
10	375



6. The y -axis of a graph shows the number of tickets sold for a circus. The x -axis shows the number of hours the circus tickets were on sale. What does the slope of the graph tell you?

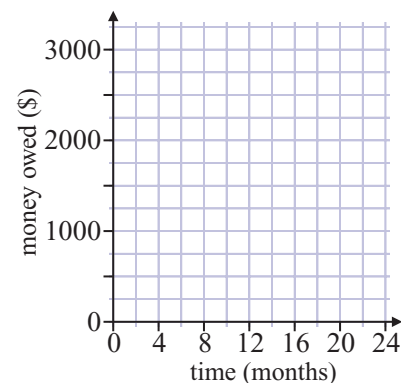


A boat travels at an average speed of 32 miles per hour. At that speed, how far could the boat travel in 2.5 hours? Create a graph to find the answer.

8. The Vitales bought a new bedroom set costing them \$3125.
- a. The Vitales chose to pay off the bedroom set in 24 equal monthly payments. Draw a graph to represent this on the grid provided.

- b. Describe the graph, including what the slope represents.

- c. If the Vitales chose to pay off the balance in 12 equal monthly payments, explain how the line graphed in part a. would change.



Lesson
4.2.3

Distance, Speed, and Time

California Standards: Algebra and Functions 4.2

1. a. Solve the formula $d = st$ for s

b. Solve the formula $d = st$ for t

2. a. A car travels 250 miles in five hours. At what rate was the car traveling?

b. How long will it take a minivan to travel 450 miles at a rate of 60 miles per hour?

Example

A climber is scaling a rock face. She moves at an average speed of 2 meters per minute. If she climbs at the same rate for 0.3 hours, how high will she have climbed?

Solution

Use the “Distance = Speed \times Time” formula to find the distance she climbs.

$$\text{Distance} = \frac{2 \text{ meters}}{\text{minute}} \times 0.3 \text{ hours}$$

$$= \frac{2 \text{ meters}}{\text{minute}} \times \left(0.3 \cancel{\text{ hours}} \times \frac{60 \text{ minutes}}{1 \cancel{\text{ hour}}} \right)$$

The time is in hours and needs to be converted into minutes so that the units match. There are 60 minutes in every hour, so multiply the hours by 60, and cancel the units down.

$$= \frac{2 \text{ meters}}{\cancel{\text{minute}}} \times (18 \cancel{\text{ minutes}}) = 36 \text{ meters}$$

Notice how the unit of time (minutes) cancels here, leaving units of distance — meters.

3. An amusement ride travels at a speed of 0.75 miles per minute.

a. How many miles does the amusement ride travel in three minutes?

b. How many miles does the amusement ride travel in 150 seconds?

c. How long in seconds does it take the amusement ride to move 1.25 miles?

4. Use the formula “distance = speed \times time” to find the missing values:

a. $d = 551$ miles $s = 58$ miles per hour $t =$

b. $s = 34$ miles per hour $t = 1.4$ hours $d =$

c. $d = 180$ kilometers $s = 40$ kilometers per hour $t =$

d. $t = 3.8$ hours $d = 114$ kilometers $s =$

5. The table shows Rosemary's photo printer's print time for an 8 in. by 10 in. color picture.

Quality	Time
Draft	3.5 minutes
Normal	6.5 minutes
Best	18 minutes

- a. Rosemary is printing 80 pictures for a scrapbook. How many more hours will it take for the photos to be printed with the best quality versus the draft quality?

- b. Thomas's photo printer will print 8 in. by 10 in. color pictures in normal quality in $\frac{1}{5}$ of an hour. Whose printer is faster in normal quality mode, Rosemary's or Thomas's?

6. a. A freight train has been traveling between 40 and 50 miles per hour. Between what distances could the train have traveled in 4 hours?

- b. A passenger train travels at a speed of 2.5 miles per minute. How many hours will it take the passenger train to travel 975 miles?

7. a. Lexi rode his bicycle at a rate of 291 yards per minute. How far did Lexi travel in 12 minutes?

- b. Drew, Lexi's twin brother, still has training wheels on his bike. He cycles at a rate of 194 yards per minute. How long will it take Drew to travel the same distance as Lexi traveled in 12 minutes?

8. An amusement ride can hold 86 people per ride.

- a. If 1032 people had been on the ride then what is the minimum number of times the amusement ride could have run?

- b. If the amusement ride had run eight times and the ride was full each time then how many people would have ridden the ride?

- c. The amusement ride had run four times. Two of the four times the ride was half full and the other two times the ride was full. How many people rode the ride?

9. Ami's DSL internet downloads at a rate of 1.5 megabits per second. Jane's cable internet downloads at a rate of 3 megabits per second.

- a. How many more megabits can be downloaded in three minutes using Jane's cable internet than Ami's DSL internet?

- b. How many more megabits can be downloaded in eight minutes using Jane's cable internet than Ami's DSL internet?

Lesson
4.2.4

Direct Variation

California Standards: Algebra and Functions 4.2

Example

If x and y are in direct variation, and $x = 6$ when $y = 3$, find x when $y = 7$.

Solution

First find the constant of proportionality:

$$k = \frac{x}{y} = \frac{6}{3} = 2$$

The formula can be rearranged to $x = ky$.

Substitute the value for k and the new value for y into the rearranged formula.

$$x = ky = 2 \times 7 = 14$$

So, when $y = 7$, $x = 14$.

1. Given that y varies directly with x , find an equation of variation for each of the following situations.

a. $y = 3$ and $x = 12$

.....

b. $y = 15$ and $x = 5$

.....

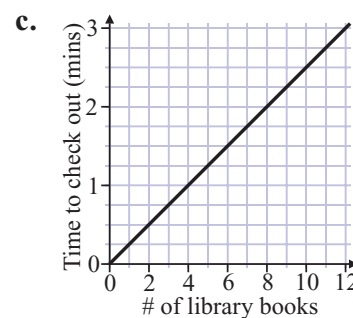
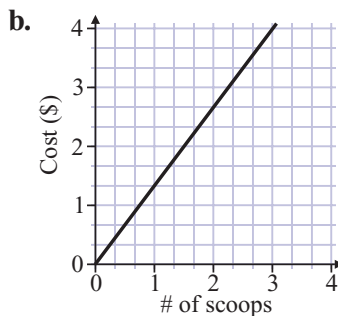
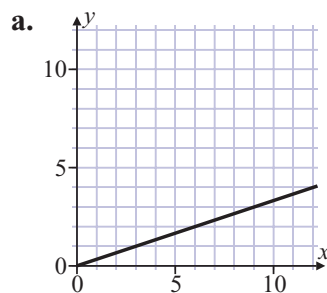
c. $y = 136$ and $x = 68$

.....

2. Explain whether $y = kx + 8$ is an equation of direct variation.

.....
.....

3. Write a direct variation equation for each graph.



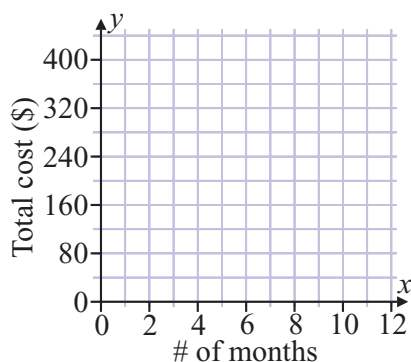
4. The Stagner family has a satellite service for \$36.50 a month.
- a. Write an equation to represent the direct variation between the number of months of service and its cost.

.....

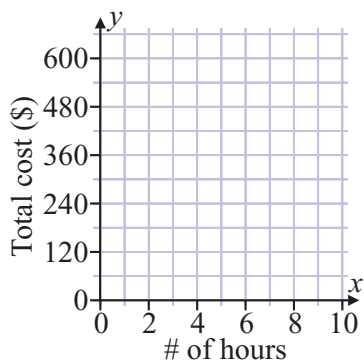
- b. Graph the direct variation.

- c. Use the graph to determine the cost of having the satellite service for 10 months.

.....



5. A personal trainer charges \$60 an hour.



- a. Write an equation to represent the direct variation between the number of hours and the cost of hiring the trainer.

.....

- b. Graph the direct variation.

- c. Use the graph to determine the cost of employing the personal trainer for 8 hours.

.....

6. Given that y and x are in direct variation, write an equation to describe their relationship for each case below.

- a. $y = 44$ when $x = 4$

- b. $y = 8$ when $x = 16$

- c. $y = 12$ when $x = 16$

7. Explain why the following situation involves a direct variation.

A window company charges \$75 per window.

.....

8. A gymnastics club charges \$16 a lesson.

- a. Write an equation to represent the total cost (in dollars) of gymnastics lessons, g , for x lessons.

.....

- b. What is the cost of 13 gymnastics lessons?

.....

Lesson
4.3.1

Converting Measures

California Standards: Measurement and Geometry 1.1

1. Give the ratios between the following pairs of units.

a. pints : cups

b. millimeters : centimeters

c. pounds : ounces

d. grams : milligrams

Example

How many centimeters are equivalent to 4.68 m?

Solution

The ratio of centimeters to meters is 100 : 1.

Let x = the number of centimeters in 4.68 meters.

Now you can use this to set up a proportion:

$$\frac{100}{1} = \frac{x}{4.68}$$

Cross-multiply and solve for x :

$$1 \times x = 100 \times 4.68$$

$$x = 468$$

So 4.68 m is equivalent to **468 cm**.

← This ratio is the same as $\frac{100}{1}$.

2. Set up and solve a proportion to find the missing value, x , in each of the metric unit conversions below.

a. 3 kilometers = x meters

b. 500 milliliters = x liters

c. 0.7 kilograms = x grams

3. Set up and solve a proportion to find the missing value, x , in each of the customary unit conversions below.

a. 16 feet = x inches

b. 48 ounces = x pounds

c. 6160 yards = x miles

4. Joaquin runs a distance of 6.25 kilometers.

a. Set up and solve a proportion to show how far this is in meters.

b. Set up and solve a proportion to show how far this is in centimeters.

5. To take part in a certain class at a horse show, a horse must be between 60 inches and 90 inches tall. Say whether each of the horses whose heights are given below could be entered into this class.

a. 6 feet

b. 2.75 yards

6. Alice is asked to convert 10 ounces to pounds. Her work is shown on the right. Explain what mistake Alice has made.

.....
.....
.....

The ratio of pounds to ounces is 1 : 16.
Let x = the number of pounds in 10 ounces.

$$\frac{1}{16} = \frac{10}{x}$$
$$1 \times x = 16 \times 10$$
$$x = 160 \text{ pounds}$$

7. Mike buys 2.8 gallons of gasoline at the gas station.

a. How many quarts of gas did he buy?

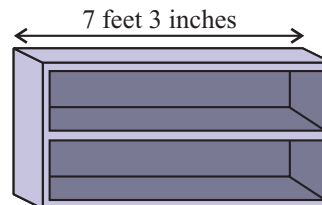
b. How many pints of gas did he buy?

8. A scone recipe calls for 0.15 liters of milk. How many milliliters is this?

.....

9. Determine the width, in inches, of the bookcase shown on the right.

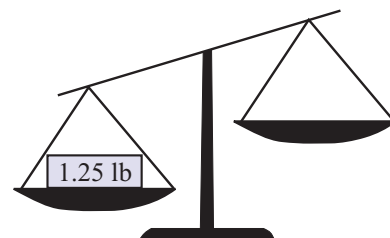
.....



10. Mia wants to make the scale shown on the right balance. She has only the five weights shown below. Which weights should she put in the right hand pan to make the scale balance?

A	B	C	D	E
2 oz	4 oz	10 oz	12 oz	14 oz

.....



Lesson
4.3.2

Converting Between Unit Systems

California Standards: Measurement and Geometry 1.1

1. Give the approximate ratios between the following pairs of units.

a. pounds : kilograms

b. yards : meters

c. liters : quarts

Example

How many kilometers are equivalent to 3 miles?

Solution

The ratio of kilometers to miles is approximately 1.6 : 1.

Let x = the number of kilometers in 3 miles.

Now you can use this to set up a proportion:

$$\frac{1.6}{1} = \frac{x}{3}$$

Cross-multiply and solve for x :

$$1 \times x = 1.6 \times 3$$

$$x = \mathbf{4.8 \text{ kilometers}}$$

2. Set up and solve a proportion to find the missing value, x , in each of the unit conversions below.

a. 4 kilograms = x pounds

b. 5 yards = x meters

3. At a local store apples cost \$3.05 per kilogram. A recipe calls for 4.4 pounds of apples. If you buy the apples at the local store, how much will they cost you?

.....

4. Elvio lives 6.08 kilometers from his school.

a. **How** far from his school does Elvio live in miles?

b. How far from his school does Elvio live in yards?

5. a. Jatoria has a jug that holds 1.514 liters of liquid. What is the volume of Jatoria's jug in gallons?
.....
- b. Matt has a very small beaker that holds just 0.02 gallons of liquid. What is the volume of Matt's jug in milliliters?
6. a. What is the formula for converting temperature in degrees Fahrenheit to temperature in degrees Celsius?
- b. What is the formula for converting temperature in degrees Celsius to temperature in degrees Fahrenheit?
7. I recorded the maximum and minimum temperature in my greenhouse over the course of one day. The maximum temperature was 77 degrees Fahrenheit and the minimum temperature was 15 degrees Celsius.
- a. What was the maximum temperature in degrees Celsius?
- b. What was the minimum temperature in degrees Fahrenheit?
8. a. Troy drives at a speed of 70 kilometers per hour, for 3 hours. How many miles will he have traveled during his 3-hour drive?
.....
- b. A snail crawls a distance of one yard. How many centimeters has the snail crawled?
.....
9. A baby bird is weighed when it hatches. Its weight is recorded as 3 grams.
- a. How many ounces did the bird weigh when it hatched?
- b. When it was fully grown, the bird weighed 0.066 pounds. How many grams did the bird's weight increase by from hatching to reaching its full size?
10. Ella, Rico, and Sati are three friends living in different cities. As part of a geography project, they each record the temperature outside their houses at the same time, on the same day.
- a. Ella says that the temperature outside her house was 18 °C. What is this in degrees Fahrenheit?
.....
- b. Rico says that the temperature outside his house was 28.4 °F. What is this in degrees Celsius?
.....
- c. Sati says that the temperature outside her house is exactly halfway between Ella's and Rico's. Give the temperature outside Sati's house in degrees Fahrenheit and in degrees Celsius.
.....

Lesson
4.3.3

Dimensional Analysis

California Standards: Algebra and Functions 4.2, Measurement and Geometry 1.3

Example

Romeo is saving up to go to science camp. He earns \$8 per hour doing yard work, and saves all the money he earns for the trip. He needs to work for 20 hours to save enough. How much money is Romeo trying to save?

Solution

Let x = the amount of money that Romeo is trying to save.

$$x = 8 \frac{\text{dollars}}{\text{hours}} \times 20 \text{ hours}$$

Do the multiplication and cancel the units

$$x = 160 \text{ dollars}$$

← The final answer is in dollars.
This is sensible, since the question asked you to find an amount of money.

1. In parts a. to e., use dimensional analysis to find the missing unit.

a. $600 \text{ words} \div 3 \text{ minutes} = 200 ?$

.....

b. $10 \text{ feet} \times 15 \text{ feet} = 150 ?$

.....

c. $10 \frac{\text{dollars}}{\text{hours}} \times 8 \text{ hours} = 80 ?$

.....

d. $60 \text{ miles} \div 40 \frac{\text{miles}}{\text{hours}} = 1.5 ?$

.....

e. $2 \frac{\text{meters}}{\text{hours}} \times 24 \frac{\text{hours}}{\text{day}} = 48 ?$

.....

2. It took Sam and two of his friends four days to build a treehouse in his backyard. How many person-days did it take to build the treehouse?

.....

3. It takes 30 person-hours to varnish a wood floor. If three people work on the floor, how long will it take them to varnish it?

.....

4. A formula says that to find the volume of a prism (in cm^3) you multiply the area of its base (in cm^2) by its height (in cm). Use dimensional analysis to check the reasonableness of the formula.

.....

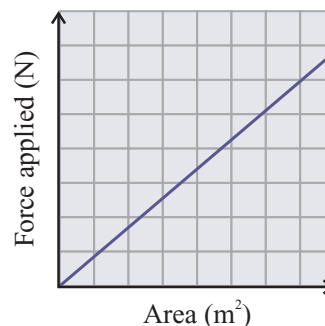
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5. I noted down that over the course of a 90-mile road trip my car used 2 gallons of gasoline. At my local garage gasoline costs \$2.80 per gallon. I am going on a 270-mile journey, and I want to fill my car with enough gas for the trip. If the car's fuel consumption stays the same, how much will the fuel cost me?
-

6. Claudia earns \$501 a week. Each week she works for 30 hours. What is her hourly rate of pay? Check your answer using dimensional analysis.
-

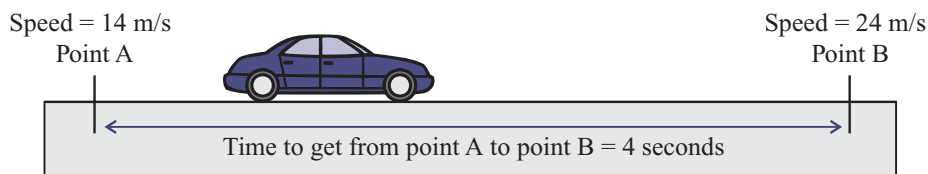
7. You are making costumes for a school play. You need 6 yards of fabric for every three costumes that you make. If you have 24 feet of fabric, how many costumes can you make? Check your answer using dimensional analysis.
-

8. Pressure is measured in units of pascals. $1 \text{ pascal} = 1 \text{ newton/m}^2$. It is suggested that the slope of the graph on the right is equal to pressure. Is this a reasonable suggestion?
-



9. A factory is testing the absorbency of a new paper towel. The testers find that it can soak up 0.6 ml of water per square inch. Given that each one is rectangular, and measures 8 inches by 6 inches, how much water can a whole paper towel absorb?
-

10. The formula used to find the acceleration of an object is $\text{Acceleration} = \text{Change in speed} \div \text{Time}$. While driving along in his car, Charlie notes his speed at two points, as shown in the diagram below.



- a. Write an equation that you could use to find the acceleration of Charlie's car.
-

- b. Solve your equation to find the acceleration of the car. Use dimensional analysis to find the correct units for your answer.
-

Lesson
4.3.4

Converting Between Units of Speed

California Standards: Measurement and Geometry 1.1

1. Use the equations below to create a conversion fraction equal to one for each pair of units.

a. 1 foot = 12 inches

b. 12 months = 1 year

c. 3600 seconds = 1 hour

d. 1 inch = 2.54 cm

2. Convert each of the following by multiplying by a conversion fraction.

a. 1.4 hours to minutes

b. 4 miles to kilometers

Example

Karen is running at a speed of 2.5 m/s. What is her running speed in km/hour?

Solution

First convert m/s into m/hour:

1 hour = 3600 seconds

Start with a conversion equation

$$\frac{3600 \text{ seconds}}{1 \text{ hour}} = \frac{1 \text{ hour}}{1 \text{ hour}} = 1$$

Divide both sides by 1 hour to make a fraction equal to one

$$\frac{2.5 \text{ meters}}{1 \text{ second}} \times \frac{3600 \text{ seconds}}{1 \text{ hour}} = (2.5 \times 3600) \frac{\text{meters}}{\text{hour}} = 9000 \frac{\text{meters}}{\text{hour}}$$

Now convert m/hour into km/hour:

1 km = 1000 m

Start with a conversion equation

$$\frac{1 \text{ km}}{1000 \text{ meters}} = \frac{1000 \text{ meters}}{1000 \text{ meters}} = 1$$

Divide both sides by 1000 meters to make a fraction equal to one

$$\frac{9000 \text{ meters}}{1 \text{ hour}} \times \frac{1 \text{ km}}{1000 \text{ meters}} = \frac{9000 \text{ meters} \times 1 \text{ km}}{1 \text{ hour} \times 1000 \text{ meters}} = \frac{9000 \text{ km}}{1000 \text{ hours}} = 9 \text{ km/hour}$$

3. If 30 miles/hour = x km/hour, then what is x ?

4. A car is traveling at a speed of 80 km/hour. What is the car's speed in miles/hour?

5. A spruce tree is growing upwards at a rate of 90 cm a year.
What is the growth rate of the tree in inches per month?

6. A raindrop is running down a window at a speed of 5 mm per second. What is the raindrop's speed in meters per minute?

7. Below are the average speeds of three athletes during an 800 m race. All three competitors were aiming to complete the race with an average speed faster than 6.4 m/s. Say whether each competitor achieved their goal.

a. 0.0068 km/s -----

b. 360 m/minute -----

c. 15 miles/hour -----

8. Amber and Mark both live four blocks away from their school, and both walk in every day.

a. Amber's average speed is 0.4 blocks per minute. Mark's is 20 blocks per hour.

Who has the higher average speed? -----

b. What is the difference, in minutes, between the time it takes Amber to reach school and the time it takes Mark to reach school? -----

9. Two runners took part in a 5000 m race. Runner 1 had an average speed of 12 km/hour over the course of the race. Runner 2 had an average speed of 225 m/minute.
Which of the two runners finished the race first?

10. In a car race, the speed of all the cars is recorded as they pass a certain point on the track. Put the three cars below in order, from slowest to fastest.

Car A: 80 miles/hour

Car B: 140 km/hour

Car C: 30 m/s

Lesson
4.4.1

Linear Inequalities

California Standards: Algebra and Functions 1.1, 4.1

1. Write each of the following inequalities in words.

a. $n > 7$

b. $m \leq 50$

c. $g \geq -5$

d. $x < -22$

2. Rewrite each of the following phrases as inequalities.

a. y is less than or equal to twelve.

b. h is greater than negative four.

c. b is less than zero.

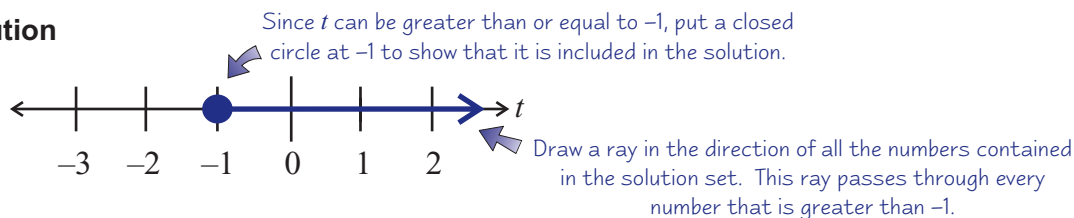
d. q is greater than or equal to negative one.

3. “ k is less than or equal to two, and the product of four and k is less than seven.”
Write a system of inequalities to represent the statement above.

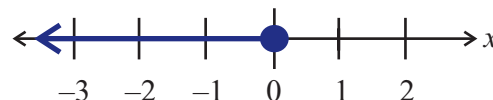
Example

Plot the inequality $t \geq -1$ on the number line.

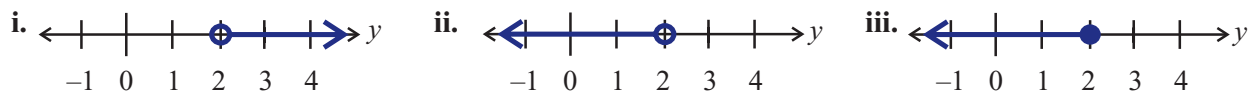
Solution



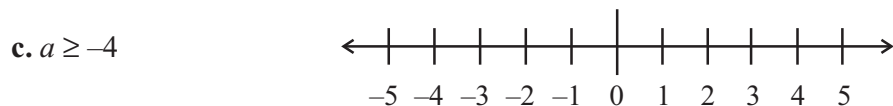
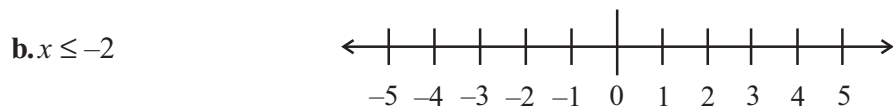
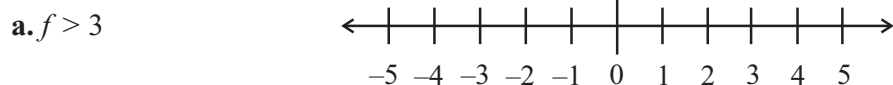
4. What is the inequality that has been plotted on the number line on the right?



5. Which of the number line plots below represents the inequality $y < 2$?



6. Plot each of these inequalities on the number line provided.



Example

Solve the inequality $x + 4 \geq 17$.

Solution

$$x + 4 \geq 17$$

$$x + 4 - 4 \geq 17 - 4$$

$$x \geq 13$$

You need to undo the operation that has been applied to the variable. In this example 4 has been added to the variable, x , so to isolate it you need to subtract 4.

Subtract 4 from each side of the inequality.
Simplify

7. Solve each of the following inequalities for the unknown.

a. $h + 10 > 20$ _____

b. $m - 5 \leq 7$ _____

c. $4 + s < 71$ _____

8. A number, d , increased by 16 is less than 22. Write an inequality to describe this, and solve it for d .

9. 55 decreased by a number, c , is greater than or equal to 14. Write an inequality to describe this, and solve it for c .

10. A school has a rule that each grade can contain no more than 190 students. The 6th grade has 31 more students than the 7th grade. Write and solve an inequality for the number of students in the 7th grade.

Lesson
4.4.2

More on Linear Inequalities

California Standards: Algebra and Functions 4.1

- Say whether you would need to reverse the sign if you applied each of the following operations to an inequality.
 - Adding -5 to both sides. _____
 - Dividing both sides by -1 . _____
 - Multiplying both sides by $-\frac{1}{2}$. _____

Example

Solve the inequality $6x \geq 18$.

Solution

$$6x \geq 18$$

$$6x \div 6 \geq 18 \div 6$$

$$x \geq 3$$

**Divide each side of the inequality by 6.
Simplify**

- Solve each of the following inequalities for the unknown.
 - $4r < 20$ _____
 - $g \div 3 \geq 25$ _____
 - $12w \leq 72$ _____
 - $y \div 4 > 8$ _____
- Which of the following is the correct solution to the inequality $w \div -4 > -5$?
 - $w > 20$
 - $w < 20$
 - $w < -20$

- Which of the following inequalities is not equivalent to the inequality $-k \div -2 \leq 8$?
 - $-k \geq -16$
 - $\frac{1}{2}k \leq 8$
 - $k \geq -16$

5. Solve each of the following inequalities for the unknown.

a. $-p < 12$

b. $d \div -11 \leq 3$

c. $-2f > -2$

d. $y \div -7 \geq -3$

6. The product of 8 and a number, n , is more than 4.
Write an inequality to describe this, and then solve it for n .

.....

7. A number, h , divided by 5 is a minimum of 13.
Write an inequality to describe this, and then solve it for h .

.....

8. The product of a number, v , and -1 is over 5.

a. Rewrite this sentence as an inequality.

b. Solve your inequality using only division.

c. Solve your inequality using only addition and subtraction.

9. A number, a , divided by negative 4 is less than 20.
Write an inequality to describe this, and then solve it for a .

.....

10. Over the summer, Cornell read more than three times the number of books that Janet read. Cornell read 15 books. Write and solve an inequality to show how many books Janet read over the summer.

.....

11. Mr. Cho's class and Ms. Chapman's class had a competition to see who could collect more boxes of paper to recycle. Mr. Cho's class collected twice as many as Ms. Chapman's class. Ms. Chapman's class collected more than 50 boxes of paper. Write and solve an inequality to show how many boxes of paper Mr. Cho's class collected.

.....

Lesson
4.4.3

Solving Two-Step Inequalities

California Standards: Algebra and Functions 4.1

Example

Solve the inequality $-2x + 4 < 8$.

Solution

$$-2x + 4 < 8$$

$$-2x + 4 - 4 < 8 - 4$$

$$-2x < 4$$

$$-2x \div -2 > 4 \div -2$$

$$x > -2$$

Subtract 4 from each side of the inequality.

**Divide each side of the inequality by -2 .
Simplify**

Undo the operations in the opposite order to the way that they were done.

Don't forget to reverse the inequality sign when you multiply or divide by a negative number.

1. Solve each of the following inequalities for the unknown.

a. $4x + 5 < 21$

b. $3b - 12 \geq -6$

c. $(g \div 3) - 13 \leq -1$

2. Jody is earning money to buy a bicycle by washing cars. She charges \$8 a car. She spends \$11 buying supplies, and the bicycle she wants costs \$125.

a. Which of the inequalities below represents the number of cars, c , that Jody must wash in order to earn enough to buy her bike?

i. $8c - 11 \geq 125$

ii. $8c + 11 \geq 125$

iii. $8c - 11 \leq 125$

iv. $8c + 11 \leq 125$

b. Solve the inequality for the number of cars, c

3. Solve each of the following inequalities for the unknown.

a. $-a + 9 < 17$

b. $-5n - 1 \geq -6$

c. $(y \div -4) + 7 \leq -4$

4. Are these two inequalities the same? Show your work.

A. $4x + 14 < 22$

B. $22 - 4x > 14$

5. Solve the inequality $\frac{w}{3} + 9 \geq 22$.

.....

6. Solve the inequality $\frac{-a}{7} - 2 < 3$.

.....

7. Arlo sets up a booth at a farmer's market one Saturday, where he sells bags of peanuts for \$3.00 each. The booth rental fee is \$35.00, and parking costs \$10.00 for the day. Write and solve an inequality to find the minimum number of bags of peanuts that Arlo must sell to make enough to pay for his parking and booth rental.

.....

8. Taisha and Jon are cousins. Taisha is two years older than twice Jon's age. Given that both cousins are under 18, write and solve an inequality to describe Jon's age.

.....

9. Sofia and Randall are both solving the inequality $7 - 10x < 32$. Sofia says the solution is $x < -2.5$, and Randall says the solution is $x > -2.5$.

a. Solve the inequality $7 - 10x < 32$.

.....

b. Who had the correct answer, Sofia or Randall?

.....

c. What was the mistake made by the person who had the incorrect answer?

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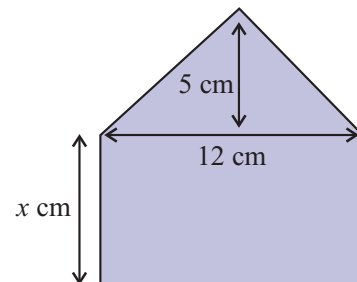
10. The area of the shape shown on the right is greater than 90 cm^2 .

a. Write an inequality using the variable x to represent the area of the shape.

.....

b. Solve your inequality for x .

.....



11. At the bakery, James buys 4 packs of rolls and 2 loaves of bread. He spends less than \$10 altogether.

a. If a pack of rolls costs \$ r , and a loaf of bread costs \$ l , write an inequality

to show how much James spent at the bakery in terms of r and l .

.....

b. Given that a loaf of bread costs \$1.28, solve your inequality to describe the cost of a pack of rolls.

.....

Lesson
5.1.1

Multiplying with Powers

California Standards: Number Sense 2.1, 2.3; Algebra and Functions 2.1

Example

Simplify $4^6 \times 4^3$.

Solution

$4^6 \times 4^3 = 4^{6+3} = 4^9$  To multiply two powers with the same base, add the exponents.

1. Say whether each statement is true or false. Correct any false answers.

a. $6^3 6^2 = 36^5$

b. $7^6 7^2 = 7^8$

c. $9^3 9^7 = 81^{10}$

d. $2^3 3^2 = 6^5$

e. $7^2 7^6 = 7^{12}$

f. $6^8 6 = 6^9$

2. Rewrite each of the following products as a single power.

a. $5^2 5^3$ b. $6^6 6^2$ c. $9^3 9^7 9^2$

d. $3^3 3$ e. $7^3 7^9$ f. $8^4 8^0$

g. $h^{11} h^7$

3. Write $6^7 6^4$ in expanded form.  Write your answer using only the 6 digit and the \times sign.

.....

4. Fill in the missing exponent for each of the following equations.

a. $3^{\dots} \dots 3^7 = 3^{11}$ b. $5^9 5^{\dots} = 5^{17}$ c. $7^3 7^{\dots} 7 = 7^{10}$

5. Simplify $x^5 y^3 (x^4 y)$

6. In 1980, computer hard disk drives had a maximum capacity of 10^7 bytes.
In 2000, computer hard disk capacity had increased to 10^{11} bytes.
By what factor had hard disk capacity increased between 1980 and 2000?
-

Write all your answers as 2^n or 3^n .

7. Simplify each expression and express it in base and exponent form.

a. 64×512

b. 16×512

c. $243 \times 19,683$

d. 2187×2187

It's helpful to make a table of powers of 2 from 2^0 to 2^{10} ,
and a table of powers of 3 from 3^0 to 3^{10} — like this:

2^0	1	3^0	1
2^1	2	3^1	3
2^2	4	3^2	9

8. Simplify $2^{2x+6}2^{3x-2}$

9. Simplify $x^3y^{-2}z^5(x^{-2}y^8z^9)$. Show your work.
-
-
-
-

10. Simplify the following expressions.

a. $n^{10}n^{12}k^5k^7$

b. $g^3h^2g^4h^5$

c. $j^6y^2j^{-3}y^5$

d. $k^{10}g^3k^{-5}gk^{-3}$

Lesson
5.1.2

Dividing with Powers

California Standards: Number Sense 2.1, 2.3

Example

What is $8^{11} \div 8^4$?

Solution

$$8^{11} \div 8^4 = 8^{11-4} = 8^7.$$



To divide powers with the same bases, subtract the second exponent from the first.

1. Say whether each statement is true or false. Correct any false answers.

a. $7^3 \div 7^2 = 7$

b. $4^8 \div 4^2 = 16^6$

c. $9^8 \div 9^5 = 9^{13}$

d. $2^9 \div 4^3 = 8^2$

e. $7^8 \div 7^3 = 7^5$

f. $6^8 \div 6 = 6^7$

2. Simplify the following expressions. Give your answer as a single power in base and exponent form.

a. $6^8 \div 6^2$ b. $8^6 \div 8^2$ c. $7^3 \div 7$

d. $2^5 \div 2^2$ e. $9^{10} \div 9^7$ f. $4^3 4^8 \div 4^9$

g. $x^8 \div x^3$

3. Simplify $5^7 \div 5^3$ and write it in expanded form. Write your answer using only the 5 digit and the \times sign.


.....

4. Fill in the missing exponent for each of the following equations.

a. $8^{\dots} \div 8^3 = 8^2$ b. $4^{12} \div 4^{\dots} = 4^8$ c. $7 \times 7^{\dots} \div 7^3 = 7^4$

5. Simplify $4^n \div 4^3$

6. Simplify $5^7 \div 5^2 n^4$

7. Simplify each expression and express it in base and exponent form.  There are powers of 10, powers of 2, and powers of 3 in this question.

a. $100,000 \div 1000$

b. $1024 \div 32$

c. $512 \div 256$


d. $6561 \div 729$

8. Simplify $x^6 y^{10} z^5 \div x^{-2}$. Show your work.

.....
.....
.....

9. A googol is 10^{100} . A googolplex is $10^{(10^{100})}$ — that's ten to the power of a googol.
What is a googolplex divided by a googol? Give your answer in base and exponent form.

.....

 Be careful — you might not be able to simplify the answer as much as you'd like.

10. Simplify the following expressions. Give your answers in base and exponent form.

a. $3^{15} \div 243$

b. $10^z \div 100$

c. $256 \div 2^a$

11. Use the division and multiplication of powers rules to show that $a^x \div (a^y \times a^z) = (a^x \div a^y) \div a^z$.

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Lesson

5.1.3

Fractions with Powers

California Standards: Number Sense 2.1, 2.3

Example

Simplify $\left(\frac{1}{4}\right)^4 \div \left(\frac{4}{5}\right)^2$.

Solution

$$\left(\frac{1}{4}\right)^4 \div \left(\frac{4}{5}\right)^2 = \frac{1^4}{4^4} \div \frac{4^2}{5^2} = \frac{1^4}{4^4} \times \frac{5^2}{4^2} = \frac{1^4 \times 5^2}{4^4 \times 4^2} = \frac{1 \times 5^2}{4^4 \times 4^2} = \frac{1 \times 5^2}{4^{4+2}} = \frac{5^2}{4^6}$$

1. Simplify each of the following expressions.

a. $\left(\frac{1}{3}\right)^4 \times \left(\frac{1}{3}\right)^6$ b. $\left(-\frac{3}{5}\right)^{11} \times \left(\frac{3}{5}\right)^{13}$ c. $\left(\frac{x}{y}\right)^7 \times \left(\frac{x}{y}\right)^{12}$

2. Simplify each of the following expressions.

a. $\left(\frac{1}{5}\right)^5 \div \left(\frac{1}{5}\right)^2$ b. $\left(\frac{7}{8}\right)^4 \div \left(\frac{7}{8}\right)^3$ c. $\left(\frac{x}{y}\right)^{10} \div \left(\frac{x}{y}\right)^6$

3. Simplify each of the following expressions as far as possible

a. $\left(\frac{1}{3}\right)^4 \times \left(\frac{2}{3}\right)^3$ b. $\left(\frac{1}{2}\right)^4 \div \left(\frac{2}{5}\right)^7$ c. $\left(\frac{3}{5}\right)^4 \times \frac{1}{25}$
d. $\left(\frac{2}{3}\right)^5 \div \frac{9}{16}$ e. $\frac{256}{81} \div \left(\frac{3}{4}\right)^8$

4. Chelsea wrote $\left(\frac{2}{5}\right)^7 \div \left(\frac{2}{5}\right)^6 = \left(\frac{2}{5}\right)^{13}$. Was she correct? Explain your answer.

.....

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5. Marco wrote $\left(\frac{9}{7}\right)^3 \times \left(\frac{5}{7}\right)^4 = \frac{9^3}{7^3} \times \frac{5^4}{7^4} = \frac{45^7}{49^7}$. Is he correct? Explain your answer.

.....

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6. Simplify $\left(\frac{6}{n}\right)^{10} \div \left(\frac{n}{12}\right)^5$

7. Simplify each of the following expressions. Express the numerators and denominators of your answers in base and exponent form.

a. $\frac{10}{9} \times \frac{100}{27}$ b. $\frac{512}{n} \times \frac{64}{n^4}$ c. $\frac{16}{5} \div \frac{125}{8}$

8. Simplify each of the following expressions as far as possible.

a. $\left(\frac{5}{2}\right)^3 \div \left(\frac{2}{6}\right)^5$ b. $\left(\frac{5}{6}\right)^2 \times \frac{25}{36}$ c. $\left(\frac{7}{3}\right)^2 \div \frac{81}{7}$

9. Simplify each of the following expressions as far as possible.

a. $\frac{5}{243} \times \left(\frac{2}{3}\right)^4$ b. $\left(\frac{2}{7}\right)^4 \div \frac{2401}{4096}$ c. $\left(\frac{2}{3}\right)^3 \div \frac{729}{2^{23}}$

10. Simplify each of the following expressions as far as possible.

a. $\left(\frac{j}{n}\right)^4 \times \left(\frac{3}{n}\right)^8$ b. $\frac{k^9}{k^6} \div \left(\frac{n}{k}\right)^2$ c. $\left(\frac{z}{n}\right)^2 \times \frac{n^6}{n^4}$

Lesson
5.2.1

Negative and Zero Exponents

California Standards: Number Sense 2.1, Algebra and Functions 2.1

Example

Simplify each of the following powers.

a. 1803^0

b. 6^{-2}

Solution

a. Any number other than 0 raised to the power 0 is 1. So $1803^0 = 1$.

b. $6^{-2} = \frac{1}{6^2}$

1. If you divide 10^n by 100 then what happens to the exponent?

2. Simplify each of the following expressions.

a. 8^0

b. $\left(\frac{1}{5}\right)^0$

c. $(-4)^0$

d. $(x + z)^0$, where $x \neq -z$

3. Simplify each of the following expressions.

a. $6^9 6^0$ b. $x^0 y^0 z^4$ c. $(4x)^0 (2)^0$

4. Rewrite 8^{-3} without a negative exponent.

5. What is 3^{-5} in expanded form?

6. If $n^x = 1$ and n is not 1, -1 , or 0, then what is x ?
.....


7. Rewrite each of the following without a negative exponent.

a. 4^{-7}

b. h^{-2}

c. $(-4)^{-10}$

d. $(q + 4)^{-6}$

 The base in this question is -4 , so your answer should have a base of -4 too.

8. Evaluate each of the following expressions.

a. $4^{-1} + \left(\frac{1}{2}\right)^2$

b. $2^3 + 4^0$

c. $9^0(6 + 2)^2$

d. $9^{-1} - 3^{-2}$

9. Why is $\frac{1}{a^{-n}} = a^n$ (where a is not equal to 0)?

.....

.....

.....

10. Rewrite each of the following expressions so that only positive exponents are used.

a. $x^{-6} - y^{-6}$

b. $\frac{1}{g^{-3}}$

c. $r^{-6} \times g^5$

d. $z^{-5} \times z^{-3}$

11. Rewrite each of the following expressions so that only negative exponents are used.

a. $\frac{1}{n^3}$

b. $d^4 \times d^0$

c. $\frac{1}{r^6} \times \frac{1}{f^9}$

d. $q^{-4} \times h^0 \times q^5$

Lesson
5.2.2

Using Negative Exponents

California Standards: Number Sense 2.1, Algebra and Functions 2.1

Example

Simplify $6^{-5} \times 6^3$.

Solution

The bases are the same, so you can use the multiplication of powers rule.

$$6^{-5} \times 6^3 = 6^{-5+3} = 6^{-2}.$$

1. Say whether the multiplication of powers rule or the division of powers rule was used in each of the following problems.

a. $8^9 \div 8^{-2}$
 $= 8^{9-(-2)}$
 $= 8^{9+2}$
 $= 8^{11}$

b. $g^{-4}g^8$
 $= g^{-4+8}$
 $= g^4$

2. Write the expanded form of 6^{-7} .

3. Simplify $\frac{18^{25}}{18^{-11}}$.

4. Fill in the missing exponents in each of the following equations.

a. $6^{-6} \times 6^{\dots} = 6^2$

b. $r^{-4}r^{-6}r^{\dots} = 1$

c. $t^8t^4 \div t^{\dots} = t^2$

d. $w^{-9} \times w^{\dots} \div w^{12} = \frac{1}{w^{29}}$

5. Is -9^3 equal to $\frac{1}{-9^3}$, $\frac{1}{9^3}$, or neither? Explain your answer.

6. Simplify v^5v^{-7} . Show your work.

.....

.....

7. Simplify $\frac{4^{-7}z^7}{4^{4+1}z^3}$. Express your answer using positive exponents.

8. Show how to solve $m^3m^{-4} \div m^{-4}$ using:

a. the multiplication of powers rule, then the division of powers rule.

.....

.....

b. the division of powers rule, then the multiplication of powers rule.

.....

.....

9. Simplify each of the following expressions. Give answers with no negative exponents.

a. t^4t^{-4} b. $7^{20} \div 7^{-20}$

c. $10^{-2}10^8$ d. $10^5 \div 10^{-3}$

e. $\frac{(q-r)^{-3}}{(q-r)^{-5}}$ f. $(a+7)^{-3}(a+7)$

g. $\frac{(c-3t)^{-4}}{(c-3t)^0}$

10. Multiply the fractions below and simplify the answers, writing them with no negative exponents.

a. $\frac{5^4}{7^{-6}} \times \frac{5}{7^{12}}$

b. $\frac{2^9}{3} \times \frac{1}{2^{-7}}$

c. $\frac{d^{-7}}{d} \times \frac{1}{d^{-2}}$

d. $\frac{4^2}{q^3} \times \frac{q^{-4}}{4^{-6}}$

Lesson
5.2.3

Scientific Notation

California Standards: Number Sense 1.1

Example

Write 4,500,000,000 in scientific notation.

Solution

$$4,500,000,000 = 4.5 \times 1,000,000,000 = 4.5 \times 10^9.$$

1. Write each of the following numbers in numerical form.

a. 4.7×10^5

b. 8×10^8

c. 2.003×10^{10}

2. Write each of the following numbers in scientific notation.

a. 67,000,000,000,000

b. 5,020,000,000

c. 400,000,000

d. 49,000

Don't miss out this zero.

3. Fill in the missing parts in the following equations.

a. $5.2 \times 10^{\text{.....}} = 520,000,000,000$

b. $\text{.....} \times 10^7 = 40,700,000$

c. $3.4 \times \text{.....}^{12} = 3,400,000,000,000$

4. An aerospace company has a total revenue of \$62.3 billion. Express this number in scientific notation.

.....

5. How many zeros are there in the numeric form of 8.2×10^{20} ? Explain your answer.

.....

.....

.....

6. Write the number 4.03×10^{-8} in numeric form.

7. Write each of the following numbers in scientific notation.

a. 0.000603 b. 0.000000000007

c. 0.000000000000056 d. 0.000000005

8. Write each of the following numbers in numeric form.

a. 3×10^{-7} b. 9.3×10^{-6}

c. 1.07×10^{-12}

9. Evaluate each expression and write your answer in scientific notation.

 This is like $\frac{1.6}{4} \times \frac{10^8}{10^{-3}}$

a. $\frac{1.6 \times 10^8}{4 \times 10^{-3}}$ b. $\frac{9 \times 10^{-3}}{3 \times 10^7}$

c. $(5 \times 10^3)(3 \times 10^8)$ d. $(3.2 \times 10^{-2})(2 \times 10^6)$

10. Evaluate each expression and write your answer in scientific notation.

a. $0.03 + 2 \times 10^{-2}$ b. $(2.3 \times 10^4) \times \frac{4 \times 10^5}{1.6 \times 10^2}$

c. $(6 \times 10^4) + (2 \times 10^4)$ d. $(1.3 \times 10^4) \times (4 \times 10^6)$

Lesson

5.2.4

Comparing Numbers in Scientific Notation

California Standards: Number Sense 1.1

Example

Which number is greater, 3.4×10^6 or 7.8×10^4 ?

Solution

The exponent of 3.4×10^6 is 6 and the exponent of 7.8×10^4 is 4.
 $6 > 4$, so $3.4 \times 10^6 > 7.8 \times 10^4$ — **3.4×10^6 is the greater number.**

1. Which number in the list below is the least? Explain your answer.

 3×10^{-5} 6×10^{-9} 1×10^{-7} 3.8×10^{-6} 2.2×10^{-3}

.....

.....

2. Put the numbers below in order from least to greatest.

 7×10^5 3×10^{-5} 2×10^3 2×10^5 7×10^{-4}

.....

3. Say which of each pair of numbers is greater.

a. 6.32×10^{12} and 3.61×10^{14} b. 2.48×10^4 and 1.2×10^2 c. 9.05×10^{11} and 1.175×10^5

4. Say which of each pair of numbers is greater.

a. 4.61×10^{-8} and 9.33×10^{-5} b. 7.03×10^{-12} and 6.23×10^{-15} c. 4.9×10^{-3} and 5.1×10^{-6}

5. The table on the right shows the masses of some of the planets in our solar system.

Planet	Mass (kg)
Mercury	3.302×10^{23}
Venus	4.869×10^{24}
Mars	6.419×10^{23}
Jupiter	1.899×10^{27}

a. Which is the most massive of the planets shown?

b. Which planet has a smaller mass, Venus or Mars?

c. Put the planets in the table in order from least to most massive.

.....

.....

6. Manuel says that 3.06×10^{-7} is greater than 5.4×10^{-4} because the exponent is bigger. Is he correct? Explain your answer.

.....

.....

.....

7. The table on the right shows the approximate populations of several countries.

Country	Population
USA	3.01×10^8
Canada	3.3×10^7
India	1.1×10^9
Ethiopia	7.5×10^7
Japan	1.28×10^8
China	1.3×10^9

a. Which country has the greatest population?

b. Which country has the least population?

8. Put the numbers below in order from least to greatest.

2.3×10^{-2} 1.8×10^{-4} 4.8×10^{15} 3.6×10^9 7.4×10^{-2}

.....

9. Put the numbers below in order from least to greatest.

0.0003 2.3×10^2 1.8×10^{-9} 23.452 18 4.5×10^5

.....

10. Which is greater — 1.23×10^{-150} divided by 100, or 2.6×10^{-149} divided by 1000? Write the greater number in scientific notation.

.....

Lesson
5.3.1

Multiplying Monomials

California Standards: Algebra and Functions 1.4, 2.2

Example

Multiply together $5v^5$ and $-8v^{-2}w^3$.


Solution

Multiply the coefficients and each variable separately.

- Multiply the coefficients: $5 \times -8 = -40$
- Multiply the powers of v together: $v^5 \cdot v^{-2} = v^3$
- Multiply the powers of w together (only one expression contains w so just include that power of w in your answer): w^3

Now multiply all these results together to form your final answer.

So $5v^5 \cdot -8v^{-2}w^3 = -40v^3w^3$.

 A monomial is a number, a variable, or the product of a number and one or more variables, or the product of two or more variables.

1. Which of the following expressions are monomials?

a. $5x$

b. $8 + 3y$

c. -12

d. $6x^2 - 3x$

e. $-12m^4n^{-2}$

2. Identify the coefficients in each of the following expressions.

a. $54g$

b. $-12c^{-2}d^6$

c. wy

d. 5^3r^4

3. Multiply $-13g^6h^{-2}$ and $-3gh^{15}$

4. Evaluate each of the following multiplications.

a. $8w^2 \cdot -3w^8$

b. $10p^{-5} \cdot -10p^9$

c. $8d^4e^{-7} \cdot 5d^{-4}e^8$

d. $-11r^3t \cdot 3r^{-5}t^4$

e. $12u^{-2} \cdot 5v^{-4}$

5. Is $10r^7w^2$ the correct solution to the multiplication $(5r^4)(5r^3w^2)$? Explain your answer.

.....

.....

6. Evaluate each of the following multiplications. Give your answers using only positive exponents.

a. $8x^3 \left(\frac{x^4}{5} \right)$

b. $\left(\frac{k^8}{7} \right) (21k^{-3})$

c. $\left(\frac{9r}{3} \right) (12r^7)$

d. $\frac{1}{4}w^{-4} \cdot \frac{1}{3}w^{-2}$

7. Fill in the missing parts of each equation.

a. $9r^3t \cdot \text{.....} r^4t \text{.....} = -45r^7t^4$

b. $-12w^4 \cdot \text{.....} w \text{.....} y \text{.....} = -4wy^3$

c. $14a^{-3}c^9 \cdot \text{.....} a \text{.....} c^{-6} = \frac{56c^3}{a^9}$

8. Square each of the following monomials.

a. b^2

b. $4x^3$

c. $5a^2b^2$

9. Evaluate each of the following multiplication expressions.

a. $j^2h^7 \cdot j^3 \cdot h^{-3} \cdot j$

b. $p^4t \cdot tq \cdot qpt$

c. $3rn \cdot 12n^3 \cdot n^{-1} \cdot 0.5$

10. Which of the two monomials below can be multiplied together to make $30x^6y^8z^2$?

$3x^3y^4z$ $10x^3y^4z^2$ $5x^6y^8z^2$ $10x^3y^4z$ $6xyz$ $6x^2y^4z$

.....

Lesson
5.3.2

Dividing Monomials

California Standards: Algebra and Functions 1.4, 2.2

Example

Find $8g^4 \div 16g^{-2}$.

Solution

Treat the coefficients and the variables separately.

- Divide the coefficients: $8 \div 16 = 0.5$
- Divide the powers of g using the division of powers rule: $g^4 \div g^{-2} = g^{4 - (-2)} = g^6$

Now multiply these results together to form your final answer.

$$8g^4 \div 16g^{-2} = 0.5g^6.$$

1. Evaluate each of the following expressions.

a. $9g^8 \div 3g^5$

b. $3v^6 \div 9v^2$

c. $-6t^5w^4 \div 12t^5w^5$

d. $\frac{a^6}{5} \div 5^2a^3c^4$

e. $15w^5r^1 \div -15w^5r^1$

2. Find the result of each of the following divisions and say whether each result is a monomial.

a. $18j^3g^2 \div 6j^3g^1$

b. $5d^6r^4 \div 10d^2r$

c. $7m^6n \div 7m^8n$

d. $14g^6h^4t^7 \div 42g^9$

3. Evaluate each of the following expressions.

a. $15p^3 \div 5p$

b. $7a^5k \div 49k^6a$

c. $65z^3kb \div 13b^8kz$

d. $20wre \div e^{19}w^2$

4. What is $36k^2z^5j^4 \div (2j^2 \cdot 3k^4)$? Show your work.

5. What is $18q^5g^{-8} \div 5^2q^{-3}g^2$? Show your work.

6. Simplify each of the following expressions.

a. $2x^3 \div (4r^3 \cdot 2x)$ -----

b. $(4t^3 \div 2tp) \cdot t^3p^2$ -----

c. $15n \div (4x^3 \div 2nx)$ -----

d. $(4g^3 + 2g^3) \div 27n^3$ -----

7. Simplify each of the following expressions.

a. $\left(\frac{8x^3}{2x^2}\right) + \left(\frac{15x^7}{3x^6}\right)$ -----

b. $\left(\frac{7}{x^3}\right) + \left(\frac{10}{x^3}\right)$ -----

c. $\left(\frac{14x}{17x^2}\right) - \left(\frac{2x}{17x^2}\right) + \left(\frac{16x}{17x^2}\right)$ -----

d. $\frac{25x^{9h-7}g^{h+8}}{40x^{4h+6}g^{-11h-9}}$ -----

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Lesson
5.3.3

Powers of Monomials

California Standards: Algebra and Functions 1.4, 2.2

Example

Write $(x^4)^6$ as a power of x .

Solution

The power of a power rule says that $(a^m)^n = a^{m \cdot n}$, so:
 $(x^4)^6 = x^{4 \cdot 6} = x^{24}$.

1. a. Write $(7^2)^5$ as a multiplication.

.....

- b. Write $(7^2)^5$ as a power of 7.

2. In the expression $(5^4)^9 = 5^x$, what is the value of x ?

3. Simplify $(-4x^3y)^3$

4. Simplify each expression. Give your answers using only positive exponents.

a. $(4^2)^{-7}$

b. $(125^{64})^0$

c. $(10^{-6})^{-3}$

d. $(19^6)^{-1}$

5. Fill in the missing part of each equation.

a. $(7^{\text{.....}})^8 = 7^{-16}$

b. $[(-3)^2]^{\text{.....}} = \frac{1}{(-3)^{12}}$

c. $(w^4)^{\text{.....}} = w^{24}$

d. $(-2r)^{\text{.....}} (4r^4)^2 = -128r^{11}$

6. Simplify $6y(4x^2)^5$. Show your work.

.....

.....

7. Show that $(-2)^4$ is not equal to $-(2^4)$.

.....

.....

8. Evaluate $[(3^2)^2]^3$.

.....

9. Simplify each of the following expressions as far as possible.

a. $[(-2x)^3]^2$

b. $[(-x)^{-10}]^{-10}$

c. $-[(3x)^2]^2$

d. $-[(-3x)^3]^{-1}$



$-((3)^2)^2$ is the same as $-1 \times ((3)^2)^2$

e. $-(-1)^3(-4x^2)^{-3}$

10. Simplify each of the following expressions.

a. $[(g^b)^r]^{-2}$

b. $-[(2j^5)^z]^6$

c. $[(3^y)^u]^v$

d. $[(-2k^2)^2]^{-7}$

11. Use the formula $y = x^3 + 2$ to find an expression for y for each of the following values of x . Simplify your expressions as far as possible.

a. $x = 7y^4$

b. $x = 2u^8e^3$

c. $x = 4t^3m^2$

d. $x = 2^3b^4$

Lesson
5.3.4

Square Roots of Monomials

California Standards: Algebra and Functions 2.2

Example

What is $\sqrt{y^2}$?

Solution

$\sqrt{y^2}$ means the positive square root of y^2 . So $\sqrt{y^2} = |y|$.

1. Fill in the missing parts of each of the following sentences.

a. $5^2 = 25$, so $\sqrt{25} = \dots\dots\dots$.

b. $7^2 = 49$, so $\sqrt{49} = \dots\dots\dots$.

c. $(v^2)^2 = v^4$, so $\sqrt{v^4} = \dots\dots\dots$.

d. $\dots\dots\dots = n^8$, so $\sqrt{n^8} = n^4$.

2. Austin and Brandon have both recently been given money. Austin was given the square root of \$144 while Brandon was given the square root of \$16 and then the square root of \$81 dollars. Who has more money? How much more does he have?
-
-

3. What is $\sqrt{h^{24}}$?


4. Simplify the following expressions. Assume all the variables are positive numbers.

a. $\sqrt{121w^6}$

b. $9\sqrt{64x^{10}}$

c. $\sqrt{4x^2} \times \sqrt{100x^8}$

d. $\sqrt{225m^{20}n^6}$

 Simplify each square root then combine the two monomials.

5. What is $\sqrt{144c^4d^8}$?

6. Simplify each of the following square roots.

a. $\sqrt{21j^{16}}$

b. $\sqrt{100h^{48}}$

c. $\sqrt{169z^2}$

d. $\sqrt{42f^8g^6}$

7. a. $\sqrt{p} = 100$. What is p ?

b. $\sqrt{q} = f^9$ What is q ?

c. $\sqrt{36r} = 6w^{12}$ What is r ?

d. $(\sqrt{s})^2 = a^{12}c^{28}$ What is s ?

8. What is $\sqrt{(k^n)^2}$?

9. Mary scored the square root of $169m^2$ on a quiz. Luann scored $9|m|$ less than the square root of $361m^2$ on the same quiz. Both their scores were positive.

a. Who scored higher?

b. How far apart were their scores?

10. Simplify each of the following expressions.

a. $9\sqrt{9y^2}$

b. $\sqrt{64x^6z^2}$

c. $10\sqrt{25g^{50}}$

d. $\sqrt{13r^{16}}$

11. Simplify the $\sqrt{7g^6j^4 \times 28g^4j^{12}}$ expression as far as possible. Show your work.

.....

Lesson
5.4.1

Graphing $y = nx^2$

California Standards: Algebra and Functions 3.1, Mathematical Reasoning 2.3, 2.5

Example

On the graph of $y = x^2$, which x -values correspond to a y -value of 225?

Solution

If $y = x^2$ then $x = \pm\sqrt{y}$. So $x = \pm\sqrt{225} = \pm 15$.

So the x -values that correspond to a y -value of 225 are $x = 15$ and $x = -15$.

Every positive number has two square roots — one positive and one negative.

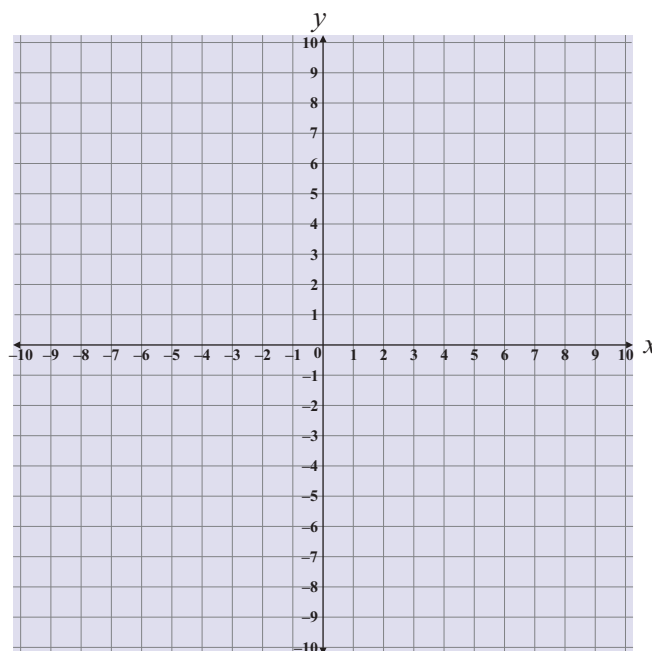
The graph of $y = x^2$ is symmetrical about the y -axis.

1. Answer the following questions for the graph $y = x^2$.

- Where is the vertex?
- What is y if $x = -7$?
- What is y if $x = 2.5$?
- What is y if $x = \frac{1}{4}$?
- Which values of x give a y -value of 81?
- Which values of x give a y -value of 900?
- Which values of x give a y -value of 0.36?
- Which values of x give a y -value of 0.25?

2. Fill in the table below and use it to graph $y = x^2$.

x	x^2
-2.5	
-1	
0	
0.5	
2	



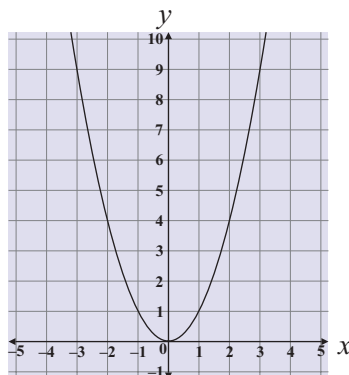
3. Plot and label each of the following points on the graph of $y = x^2$.

A (1.5, 2.25)

B (-3, 9)

C $\left(\frac{11}{5}, \frac{121}{25}\right)$

D (-1.8, 3.24)



4. $(x, 0.2)$ is on the graph of $y = nx^2$. Is $(-x, 0.2)$ on the graph? Explain your answer.

.....

.....

5. (12, 144) is on the graph of $y = x^2$. Without putting the x -value into the equation, explain how you know what the value of y will be at $x = -12$.

.....

6. Which two values of x give $y = 25$ when put into the equation $y = x^2$?

.....

7. Which graph is wider than the graph $y = x^2$, $y = 4x^2$ or $y = \frac{1}{4}x^2$? Explain your answer.

.....

.....

.....

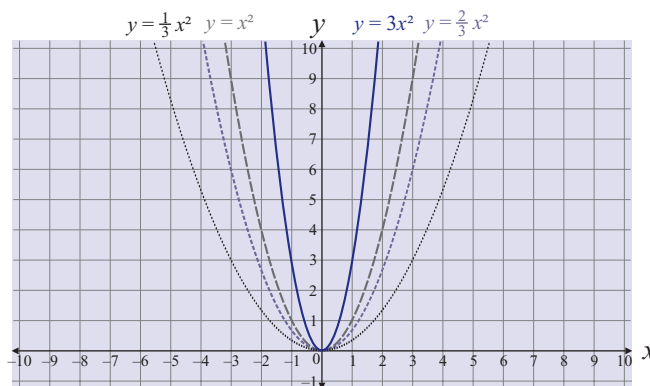
8. Say which of the graphs on the right contain each of the following points.

a. (3, 6)

b. (-6, 12)

c. (1, 3)

d. (-2, 12)



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Lesson
5.4.2

More Graphs of $y = nx^2$

California Standards: Algebra and Functions 3.1, Mathematical Reasoning 2.3

Example

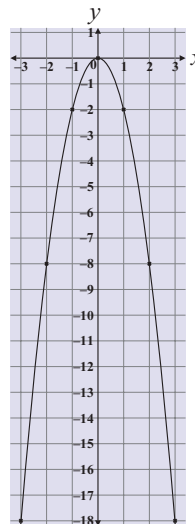
Graph $y = -2x^2$ for x between -3 and 3 .

Solution

First make a table of values.

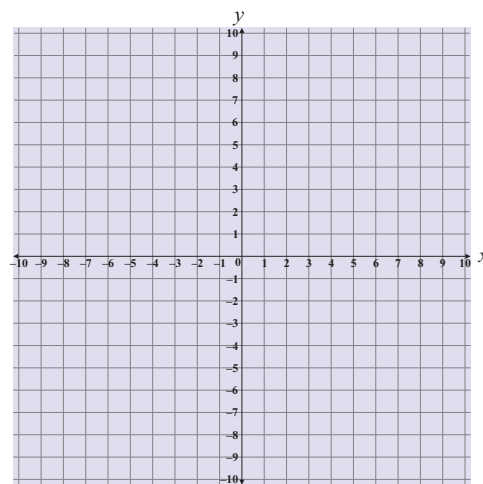
x	-3	-2	-1	0	1	2	3
$-2x^2$	-18	-8	-2	0	-2	-8	-18

Then plot the points, and join them to make the graph:



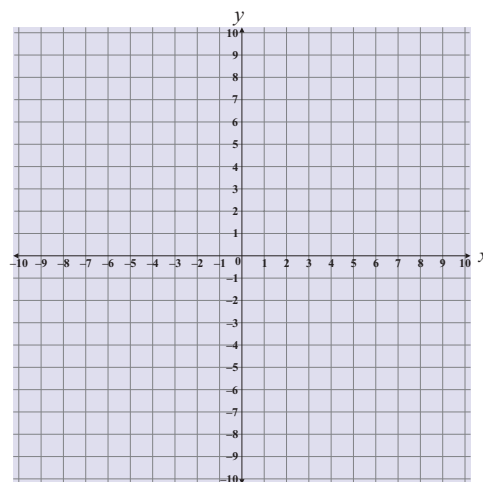
1. Graph $y = x^2$ and $y = -x^2$.

x	x^2	$-x^2$
-2.5		
-1		
0		
1		
2.5		



2. Graph $y = 0.75x^2$.

x	$0.75x^2$
-3	
-2	
-1	
0	
1	
2	
3	



3. Identify each of the following statements as either true or false.

a. $y = 4x^2$ is narrower than $y = x^2$ b. $y = -\frac{1}{5}x^2$ is narrower than $y = -x^2$

c. $y = \frac{2}{4}x^2$ is the same as $y = 0.5x^2$

d. $y = 6x^2$ is the upside-down version of $y = \frac{1}{6}x^2$

e. $y = -3x^2$ is narrower than $y = -2x^2$ f. $y = 5x^2$ is u-shaped.

g. $y = \frac{3}{4}x^2$ is symmetric about the x -axis.

4. a. On the graph of $y = 8x^2$, what is y when $x = -7$?

b. On the graph of $y = 0.95x^2$, what is y when $x = 2$?

c. On the graph of $y = \frac{2}{5}x^2$, what is y when $x = 10$?

d. On the graph of $y = -3x^2$, which values of x give $y = -75$?

e. Which values of x give $y = 6$ on the graph of $y = \frac{1}{6}x^2$?

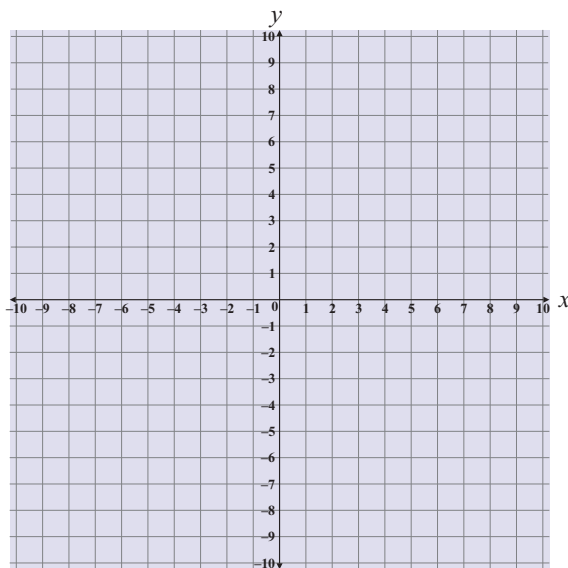
5. a. Graph $y = x^2$.

b. Graph $x = y^2$.

An easy way to graph $x = y^2$ is to turn the graph paper on its side so that the y -axis is where the x -axis would normally be and the x -axis is where the y -axis would normally be.

c. Identify each graph's vertex.

d. Could you describe either graph as upside-down u-shaped? If so, then which?



6. The point $(5, 125)$ is on the graph of $y = 5x^2$. Without doing any calculations, what is the y -coordinate on the graph of $y = -5x^2$ when $x = 5$?

7. Without plotting any points, describe what the graph of $y = -50x^2$ would look like.

Lesson 5.4.3

Graphing $y = nx^3$

California Standards: Algebra and Functions 3.1, Mathematical Reasoning 2.3

Example

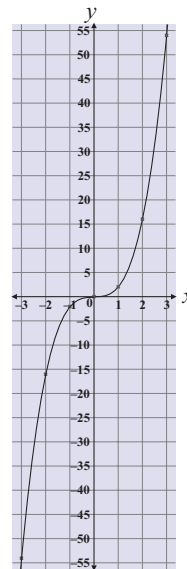
Graph $y = 2x^3$ for x between -3 and 3 .

Solution

First make a table of values.

x	-3	-2	-1	0	1	2	3
$2x^3$	-54	-16	-2	0	2	16	54

Then plot the points, and join them to make the graph:



- The statements below are all about the graph of $y = x^3$. Say whether each statement is true or false.
 - The vertex is at $(0, 0)$
 - The y -axis is the axis of symmetry.
 - If $x = -4$, then $y = 64$
 - If $y = -125$, then $x = -5$
- Do the y -values increase or decrease between $x = -10$ and $x = 0$ on the graph of $y = x^2$?
.....
 - Do the y -values increase or decrease between $x = 0$ and $x = 10$ on the graph of $y = x^2$?
.....
 - Do the y -values increase or decrease between $x = -10$ and $x = 0$ on the graph of $y = x^3$?
.....
 - Do the y -values increase or decrease between $x = 0$ and $x = 10$ on the graph of $y = x^3$?
.....
- What is y when $x = -5$ on the graph of $y = x^3$?
- How many values of x give the same value of y on a $y = nx^3$ graph, (assuming that $n \neq 0$)?

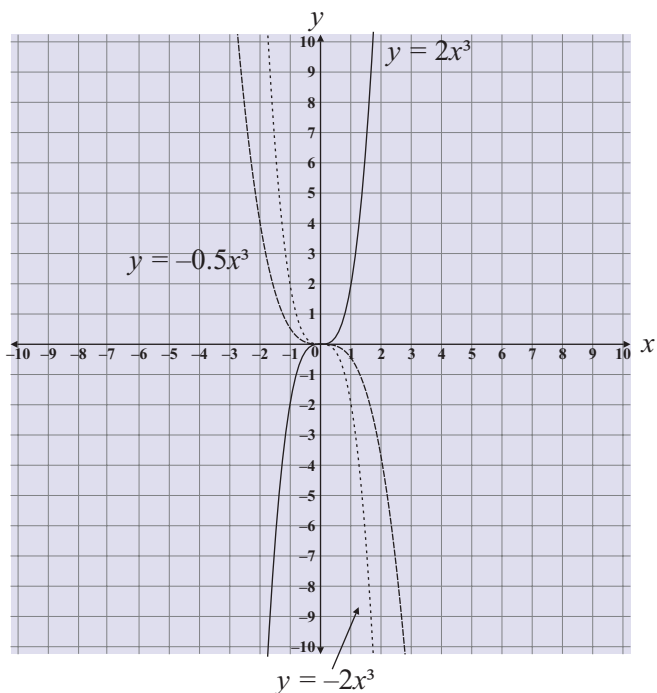
5. a. On the graph of $y = 0.5x^3$, what is y when $x = -5$?
- b. On the graph of $y = -x^3$, what is y when $x = 4$?
- c. On the graph of $y = \frac{2}{3}x^3$, what is y when $x = 1$?
- d. On the graph of $y = -2x^3$, what is x when $y = -54$?
- e. On the graph of $y = 5x^3$, what is x when $y = -5$?
6. $(7, 343)$ is on the graph of $y = x^3$. Without doing any calculations, what is y when $x = -7$?

7. Which graph contains each point?

- a. $(1, 2)$
- b. $(-1, 2)$
- c. $(1, -0.5)$
- d. $(-2, 4)$



You could plot the ordered pairs and see which of the graphs each point is on.
(Or you could substitute the x -values into each equation to see if the y -value matches.)



8. What would you have to do to a graph of $y = x^3$ so that it became a graph of $y = -x^3$.
.....
9. Say whether each of the following points is on the graph of $y = 6x^3$.
- a. $(-2, 48)$
- b. $(2, 48)$
- c. $(6, 1296)$
- d. $\left(\frac{1}{3}, \frac{2}{9}\right)$
- e. $(-0.5, 0.75)$

Lesson
6.1.1

Median and Range

California Standards: Statistics, Data Analysis, and Probability 1.3

Example

Find the median of each data set below.

- a. {26, 37, 18, 37, 19}
- b. {22, 22, 11, 20}

Solution

- a. Order the data set: {18, 19, 26, 37, 37}
The set has five values. This is an odd number, so the median is the middle value, which is **26**.
- b. Order the data set: {11, 20, 22, 22}
The set has four values. This is an even number, so the median is the average of the two middle values, which are 20 and 22. $(20 + 22) \div 2 = 21$. So the median is **21**.

1. Find the median of each of the following data sets.

a. {560, 780, 499, 506, 687, 730}

.....

b. {5.6, 4.3, 5.0, 6.8, 6.8, 5.0, 4.9}

.....

c. {18, 19, 7, 4, 19, 11, 12}

.....

d. {0.6, 0.06, 1.6, 0.7, 0.9, 0.1}

.....

2. The median of the data set {24, 35, 30, 12, 40, x } is 28. What is the value of x ?

.....

3. Four cranes of different heights are being used to construct a building. The heights of the cranes are 123 feet, 265 feet, 267 feet, and 208 feet. What is the median height of these four cranes?

.....

4. Over the past five years, the median attendance at the October Music Festival was 120,000.
a. What does this tell you about the attendance at the October Music Festival three years ago?

.....

- b. Sadie says that the fact that the median attendance was 120,000 means that at least 100,000 people attended the festival each year. Is she right? Explain your answer.

.....

.....

Example

Find the range of the data set below.

{64, 99, 54, 87, 79}

Solution

The minimum value in the data set is 54. The maximum value is 99.

The range is the difference between the maximum and minimum values.

So the range of the data set is $99 - 54 = 45$

5. Find the range of each of the two data sets below.

a. {59, 60, 72, 83, 40, 84}

b. {6.5, 8.9, 4.9, 10.0, 6.6, 11.7, 5.3}

6. The last five houses to be sold on a street went for the following amounts:

{\$185,000, \$222,000, \$142,500, \$199,000, \$251,500}

What is the range of the house prices?

7. This data set shows the number of games played by a major league baseball player during his first six seasons as a professional: {161, 157, 157, 154, 155, 142}

a. What is the median number of games played by the player in his first six seasons?

b. What is the range of the number of games played by the player in his first six seasons?

c. In his seventh season he played 164 games. What is the range of the number of games played by the player in his first seven seasons?

8. The table on the right shows the land area of six states in miles².

a. What is the range of the data set consisting of California, Indiana, New York, and Kansas?

b. What is the median of the data set consisting of California, Indiana, New York, and Kansas?

c. What are the new median and range if you add Alaska to the set?

d. What are the new median and range if you add Alaska and Rhode Island to the set?

State	Land area (miles ²)
Indiana	35,867
Rhode Island	1,045
California	155,959
New York	47,214
Alaska	571,951
Kansas	81,816

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Lesson
6.1.2

Box-and-Whisker Plots

California Standards: Statistics, Data Analysis, and Probability 1.1, 1.3

Example

Construct a box-and-whisker plot from the following data set:

{11, 12, 23, 23, 25, 27, 35, 43, 44, 45, 48}

Solution

Find the five key values for the set that you need to draw the box-and-whisker plot:

The minimum and maximum are 11 and 48.

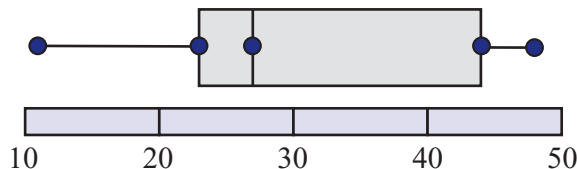
There are eleven values in the data set.

The median value must be the sixth value, which is 27.

The lower quartile must be the third value, which is 23.

The upper quartile must be the ninth value, which is 44.

Now plot the data points, and make the box and whisker plot:



1. The data set below shows the number of goals scored by some members of a soccer team in a season.

{5, 8, 14, 20, 23, 25, 28}

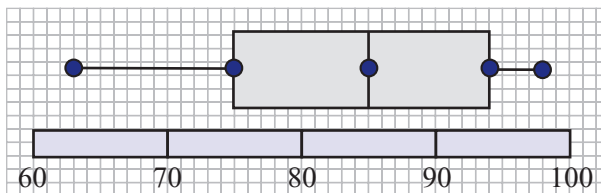
a. What are the maximum and minimum values?

b. What is the median of the data set?

c. What is the upper quartile of the data set?

d. What is the lower quartile of the data set?

2. The box-and-whisker plot below summarizes the average test scores of Mr. Turner's math class.



a. What are the maximum and minimum values?

b. What is the median of the data set?

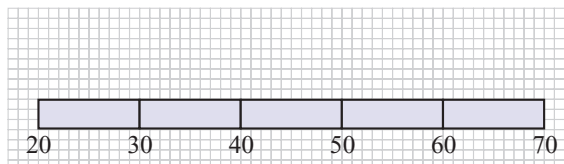
c. What does the plot tell you about the number of values in the data set?

3. The data set below shows the ages of the people attending an evening class.

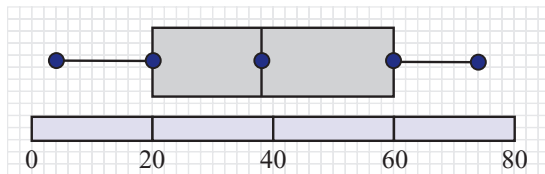
{22, 26, 27, 28, 32, 36, 42, 42, 48, 55, 56, 56, 59, 68, 69}

- How many values are there in the data set?
- What are the maximum and minimum values?
- What is the median of the data set?
- What is the upper quartile of the data set?
- What is the lower quartile of the data set?

- Use your answers to parts **a.** to **e.** to draw a box-and-whisker plot of the data on the grid on the right. A scale has been provided to help you.



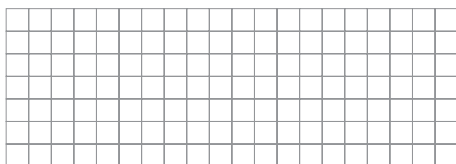
4. Based on the box-and-whisker plot shown below, say whether each statement is true or false.



- The range of the data set is 74.
- The median of the data set is 40.
- The data set contains no values higher than 60.
- The data set contains no values lower than 4.

5. Draw a box-and-whisker plot of the following data set on the grid below.

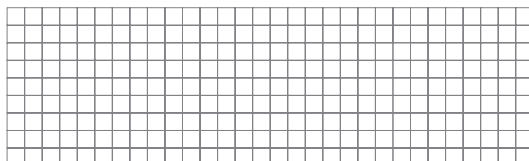
{90, 70, 85, 75, 95, 100, 70, 85, 75, 105}



6. Molly is growing pea plants. She plants 14 seeds, and after three weeks she measures the height of all her seedlings to check on their progress. These were the heights of her plants in centimeters:

{12.8, 12.6, 12.4, 12.3, 12.0, 11.7, 11.6, 11.3, 11.2, 11.0, 10.8, 10.6, 10.4, 10.2}

Draw a box-and-whisker plot to show Molly's data on the grid below.



Lesson

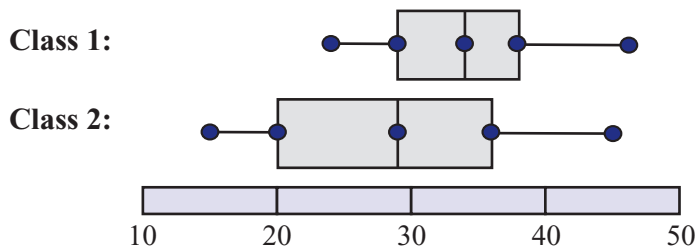
6.1.3

More on Box-and-Whisker Plots

California Standards: Statistics, Data Analysis, and Probability 1.1

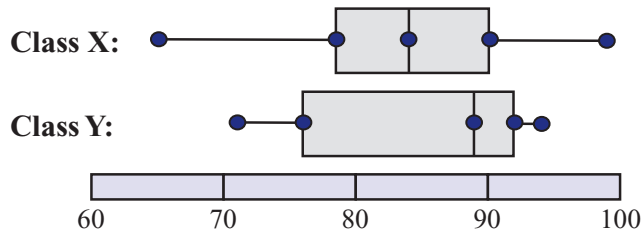
Example

The box-and-whisker plots below show the ages of people attending two karate classes. What do they tell you about the ages of the people attending the two classes?

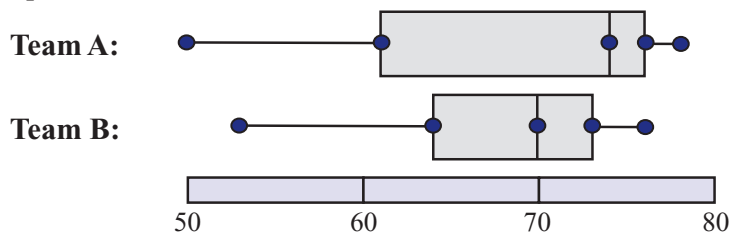
**Solution**

The students attending class 1 are typically older than those attending class 2, because it has a higher median. The middle 50% of ages are more spread out in class 2 than they are in class 1, so you can say that the age of people attending class 1 is more concentrated around the median age. The maximum and minimum ages are further apart in class 2 than in class 1, so class 2 has a greater age range.

1. The test scores of two math classes are summarized by the double box-and-whisker plot below.



- a. Which class has a higher median score?
- b. Which class has a greater range of scores?
2. The points earned during each game of the season for two basketball teams are summarized on the box-and-whisker plots below.



- a. Which team had typically lower scores across the season?
- b. Which team had the greater range of scores across the season?
- c. Which team scored 74 points or more in around half of its games?

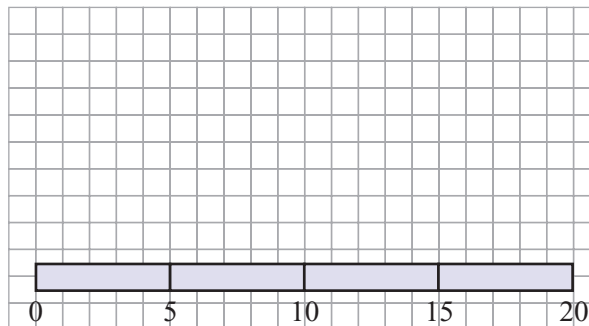
3. Tyler asked all the members of the school band and the school orchestra how many hours a week they practiced their music. This is the data he gathered:

Band {6, 10, 14, 8, 9, 15, 9, 8, 11, 7, 10, 11, 13}

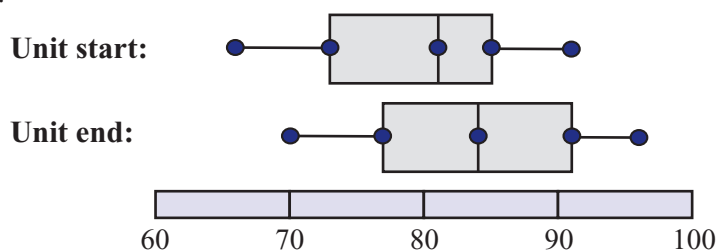
Orchestra {10, 16, 13, 9, 11, 12, 12, 14, 13, 15, 13, 14}

- a. Plot the data as a double box-and-whisker plot on the grid on the right.

- b. Compare the data using your plots.

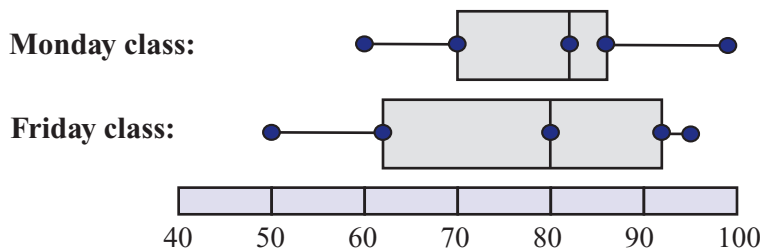


4. Mrs. Kim gives her sixth grade math group a test at the beginning and end of each unit. The two box-and-whisker plots below summarize the unit start and unit end test scores for the first unit of the year.



What do the box-and-whisker plots tell you about the difference between the unit start and unit end test scores?

5. Mr. Ellis made the box-and-whisker plots below to compare the grade averages of his Monday and Friday geography classes.



- a. If a mark of 70 is considered as a pass, which class had the highest proportion of students pass?

- b. Approximately what percentage of students in the Friday class scored 80 or above?

Lesson
6.1.4

Stem-and-Leaf Plots

California Standards: Statistics, Data Analysis, and Probability 1.1

Example

The stem-and-leaf plot below shows the first-round scores of the 23 golfers playing in a tournament.

```

6   6 6 7 8 8 8 9 9
7   0 0 0 1 2 2 3 4 5 6
8   1 2 6 6 9
  
```

- What was the median score of the competitors?
- If par was 71, how many players completed the first round with a score that was under par?

Solution

- There are 23 values in the data set. The median is the middle data point, which in this case is the 12th value. The 12th value in the stem-and-leaf plot is **71**.
- If par was 71, then any player that completed the round in 70 or less was under par. There are 11 values of 70 or lower, so **11 players** were under par.

- Complete the stem-and-leaf plot below showing the test scores of Mrs. Farley's Algebra I class. The scores were as follows: {90, 72, 88, 75, 97, 77, 72, 84, 87, 95, 79, 80, 93, 95, 88, 87, 78, 69, 91}

```

6   -----
7   -----
8   -----
9   -----
  
```

- The stem-and-leaf plot below shows the lengths of all the sail boats being kept in a harbor.

```

2   2 4 5 6 6 6 8 8 9 9 9
3   0 0 1 4 4 4 4 4 5 6 6 6 8 8
4   1 5 5
  
```

- How many sail boats were being kept in the harbor?
- How many boats larger than 34 feet were being kept in the harbor?

- The stem-and-leaf plot below shows the lengths of a group of wild ducks in inches.

```

1   4 5 6 7 7 8 9 9
2   0 1 1 2 4 4 5 6
  
```

- How many ducks' lengths were measured and recorded?
- Which four lengths occur more than once?
- What is the range of the ducks' lengths?
- What is the median of the ducks' lengths?

4. The stem-and-leaf plot below shows the number of inches of rain that fell in 15 towns in March.

```

0    1 3 5 5 7 9
1    0 1 1 2 5 7 8
2    0 1

```

List all the individual data values contained in the stem-and-leaf plot.

5. The back-to-back stem-and-leaf plot below compares the test scores of Mr. Hill's two history classes.

<u>Class 1</u>						<u>Class 2</u>				
	8	7	5	3	9	3	5	7	8	
7	5	0	0	0	8	1	2	3	4	5
		9	7	5	7	4	5	5	7	9
	6	5	4	3	6	5	7	8		
		8	7	5	5					
			5		4	0				

- a. How many students in Class 1 took the test? -----
- b. Which class had a median score of 80? -----
- c. Which class had a range of 53? -----
6. The stem-and-leaf plot below shows the game scores earned by a football team over two seasons.

<u>Season 2</u>						<u>Season 1</u>				
			3		4	0				
8	5	3	3		3					
	7	1	0		2	0	2	4	7	
			7		1	2	4			
					0	6	8			

Use the stem-and-leaf plot to compare the team's season 1 performance to its season 2 performance.

7. The table below shows the number of rebounds gained in a season by players for two basketball teams.

Team A	53	20	20	29	33	16	18	27	15	15	22
Team B	44	66	46	45	21	22	17	16	29	15	19

- a. Make a back-to-back stem-and-leaf plot of the data in the box on the right.
- b. Which team had no players with rebounds in the 30s?

- c. Which team has the larger range of rebounds per player?

- d. Which team has a median of 22 rebounds?

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Lesson

6.1.5

Preparing Data to Be Analyzed

California Standards: Statistics, Data Analysis, and Probability 1.1

Example

A company that makes low-energy lightbulbs conducts a test to find how long its products typically last for. The data set below shows the time, in hours, that each lightbulb tested lasted for.

{9500, 13,000, 8000, 8500, 15,500, 5500, 9500, 10,000,
12,500, 11,000, 9000, 13,000, 12,500, 14,000, 7000}

Prepare the data set to be analyzed by finding its minimum and maximum values, its range, its median, and its upper and lower quartiles.

Solution

The highest value in the data set, which is its maximum, is **15,500 hours**.

The lowest value in the data set, which is its minimum, is **5500 hours**.

The range of the data set is its maximum minus its minimum, which is $15,500 - 5500 = 10,000$ hours.

Put the data set in order: {5500, 7000, 8000, 8500, 9000, 9500, 9500, 10,000, 11,000,
12,500, 12,500, 13,000, 13,000, 14,000, 15,500}

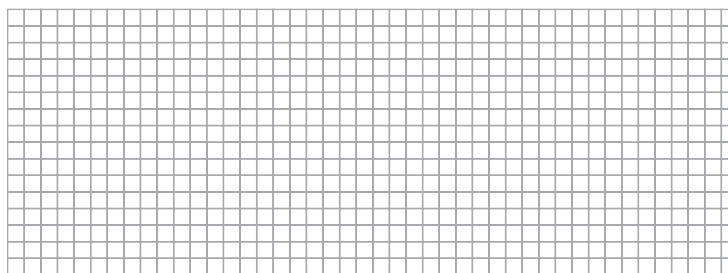
The median is the middle value of the data set. Since it has 15 values, the median is the eighth value, which is **10,000 hours**.

The lower quartile is the fourth value in the data set, which is **8500 hours**.

The upper quartile is the twelfth value in the data set, which is **13,000 hours**.

- The data set below shows the average depth of a river, in cm, every day for two weeks.
{186.3, 188.0, 189.4, 188.7, 190.3, 190.9, 194.2, 190.6, 191.7, 190.1, 187.8, 188.2, 185.7, 185.6}
 - Find the median of the data set. _____
 - Find the range of the data set. _____
 - Find the lower quartile of the data set. _____
 - Find the upper quartile of the data set. _____
- The two data sets below show the values, in dollars, of the same houses on a street in 2000 and 2005.
2000: {500,000, 550,000, 450,000, 600,000, 700,000, 650,000, 700,000, 500,000}
2005: {550,000, 605,000, 495,000, 660,000, 770,000, 800,000, 800,000, 600,000}
 - Find the maximum of both data sets. _____
 - Find the minimum of both data sets. _____
 - Find the median of both data sets. _____
 - Find the lower quartiles of both data sets. _____
 - Find the upper quartiles of both data sets. _____

3. Display the data you prepared in Exercise 2 as a double box-and-whisker plot. Use the grid below.



4. A grocer noted the amount, in dollars, spent by 20 of her customers. This is the data she collected:
 $\{20, 16, 18, 35, 33, 19, 21, 20, 22, 27, 25, 24, 24, 23, 35, 33, 17, 16, 28, 28\}$

- a. Find the minimum, maximum, median, lower quartile, and upper quartile of the data set.

- b. Show the data set on a stem-and-leaf plot.
 Draw your plot in the box on the right.



5. Mr. Marquez used the back-to-back stem-and-leaf plot shown below to display the grade averages of his first period and fourth period French classes.

<u>1st Period</u>							<u>4th Period</u>								
		5	4	2	1	1	0	9	0	0	5	5	6	8	9
8	7	5	0	0	0	0	0	8	0	0	0	5	5	5	
				6	5	0		7	0	0	5	5	5		
				7	5	0	0	6	0	5					

- a. Find the minimum, maximum, median, lower quartile, and upper quartile of the scores for each class.

- b. Plot the data on a double box-and-whisker plot using the grid below.



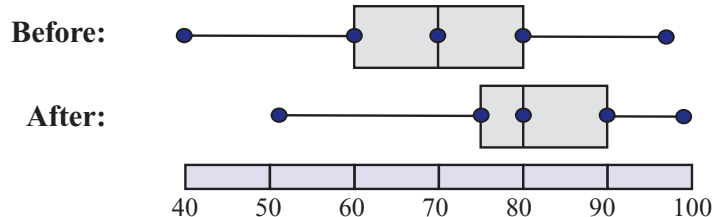
Lesson
6.1.6

Analyzing Data

California Standards: Statistics, Data Analysis, and Probability 1.1, Mathematical Reasoning 2.6

Example

A school is trying out a new science program. The box-and-whisker plots below show the grade averages of 20 students before and after being on the new program.

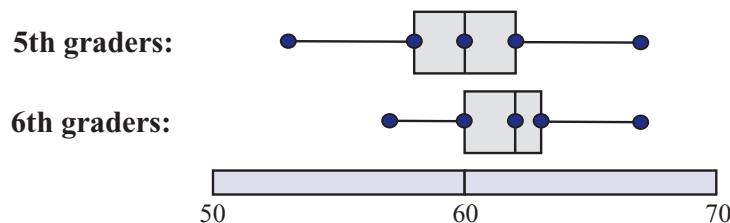


- Approximately what percent of students scored 81 or above before starting the new program?
- What percent scored 80 or above after trying the program?
- Should the school use the new program? Use the box and whisker plots to justify your answer.

Solution

- The upper quartile of the before group is around 80. So around **25%** of the students scored 80 or higher before starting the program.
- The median of the after group is around 80. So around **50%** of the students scored 80 or higher after trying the program.
- The school should use the new program. After trying it, the median and upper and lower quartiles of the scores all increased, showing that the students on the program were performing better overall.

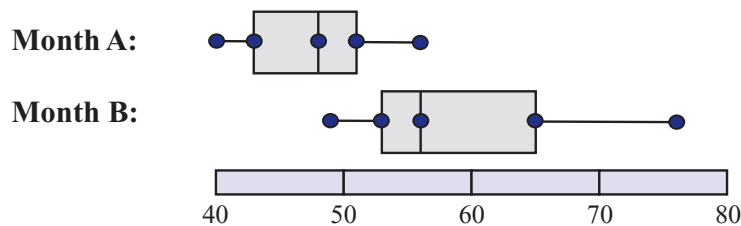
- Look at the box-and-whisker plots below. They show the heights, in inches, of 40 randomly selected fifth graders and 40 randomly selected sixth graders.



Say which of the following statements are true, and which are false, by looking at the plots.

- The range of the fifth graders' heights is larger than the range of the sixth graders' heights.
- About 50% of the sixth graders are taller than 60 inches.
- About a quarter of the fifth graders are between 58 and 60 inches tall.
- The sixth graders are typically taller than the fifth graders.

2. The box-and-whisker plots below show the daily temperatures in a California city in January and June. The temperatures shown are in degrees Fahrenheit.



Which plot do you think shows June and which shows January? Explain your answer.

.....

.....

3. The stem-and-leaf plot below shows the number of CDs owned by the students in two classes.

<u>Class A</u>		<u>Class B</u>
9 8 7 6	1	1 2
7 7 5 5	2	3 5 5
9 8 8 7	3	7 8 9 9 9
3 1	4	1 1 1 1 2 3 3 4 5
9	5	0

What conclusions can you draw from the data display?

.....

.....

4. Martin notes down the number of chilled drinks sold by eight vending machines on a cold day and on a warm day. The data he collected is shown in the table below.

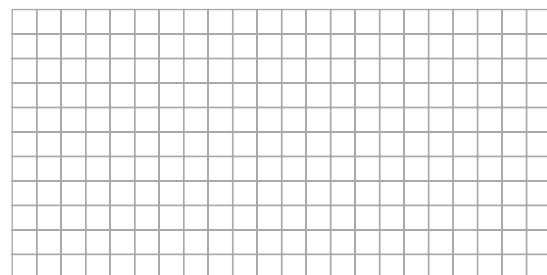
Cold day	37	37	39	32	38	39	34	33
Warm day	39	38	40	41	39	44	37	36

- a. Find the maximum, minimum, median, lower quartile, and upper quartile of the two data sets.

.....

.....

- b. Plot the data as a double box-and-whisker plot, using the grid on the right.



- c. Did the machines sell more drinks on the cold day or the warm day? Use the box-and-whisker plot to justify your answer.

.....

.....

Lesson 6.2.1

Making Scatterplots

California Standards: Statistics, Data Analysis, and Probability 1.2

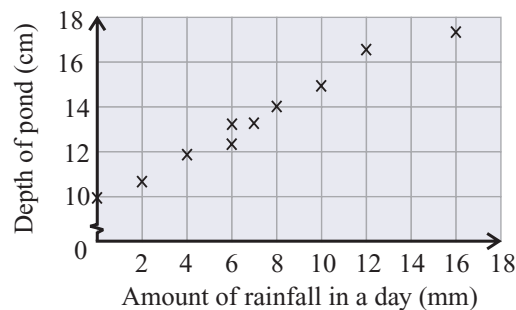
1. What data would you need to collect in order to test the conjecture:
“The more a student studies, the better the score they get on their math test”?

2. What data would you need to collect in order to test the conjecture:
“The taller a student is, the earlier in the year their birthday is likely to be”?

3. The Wilson family have a pond in their garden. Each day they note down the depth of the pond, and the amount of rainfall that day. The scatterplot below shows the data they collected over ten days.

- a. Use the scatterplot to complete this statement:
“The greater the amount of rain that fell, the
..... the depth of the Wilsons’ pond.”

- b. What was the depth of the pond on the day that
8 mm of rain fell?



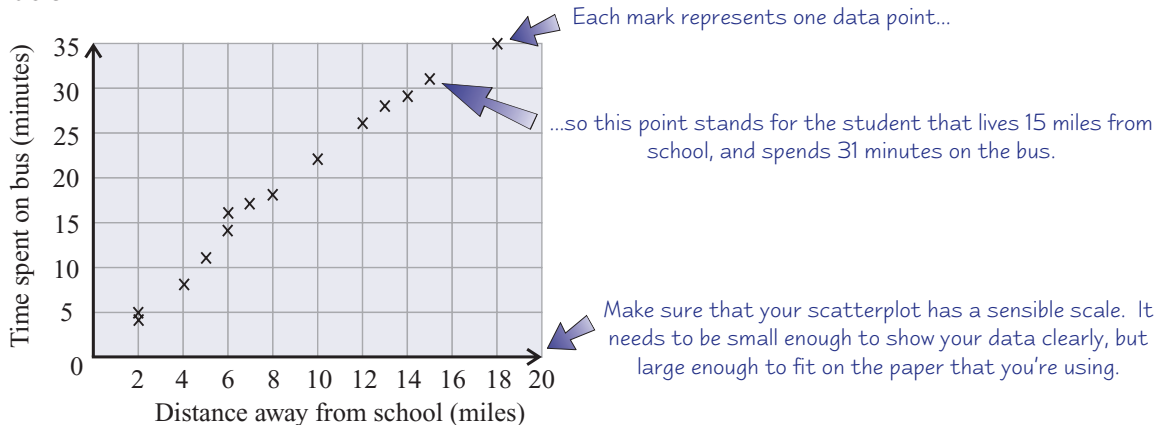
Example

Hope asks everyone on her bus how far away from school they live, and how many minutes they spend on the bus each journey. The data she collected is shown in the table below.

Distance from school (miles)	2	2	4	5	6	6	7	8	10	12	13	14	15	18
Journey Time (minutes)	4	5	8	11	14	16	17	18	22	26	28	29	31	35

Make a scatterplot of the data.

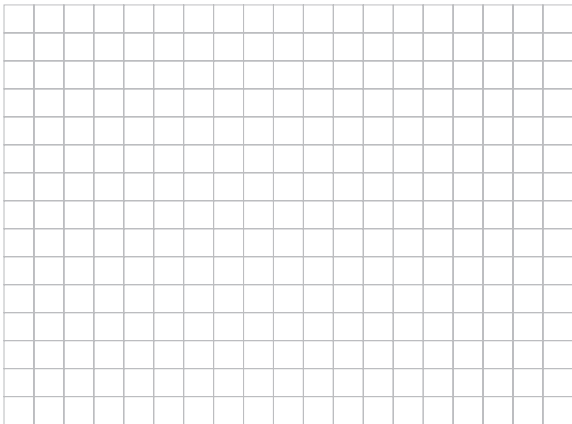
Solution




4. Write a conjecture that Hope might have been testing when she collected the data in the Example.

5. The data shown in the table below was collected to test the conjecture: “The older a child is, the larger their shoe size will be.” Draw a scatterplot of the data on the grid provided.

Age (years)	6	8	9	10	11	12	13	14
Shoe size	4	6	6	7	8	8	9	9



6. The data shown in the table below was collected to test the conjecture: “The older a person is, the more pets they are likely to own.” Draw a scatterplot of the data on the grid provided.


 The scale of your graph doesn't have to start at zero.

Age (years)	20	24	26	28	30	36
Number of pets	2	0	1	4	1	2



7. The data shown in the table below was collected to test the conjecture: “The lower the temperature is outside, the more winter coats a store will sell.”

Temperature (°F)	35	40	45	50	55	60	65	70	75
Number of coats sold	18	18	16	16	10	8	4	10	1

- a. Draw a scatterplot of the data on the grid provided.
- b. On one of the days, the store where the conjecture was tested had a sale, and sold more coats than they would have done normally.
What was the temperature on this day?



Lesson
6.2.2

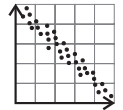
Shapes of Scatterplots

California Standards: Statistics, Data Analysis, and Probability 1.2

1. Join the correct description (blue box) and example plot to the correct type of correlation (gray box).

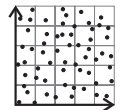
As the x -value increases, the y -value increases.

Positive correlation



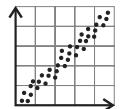
There is no obvious pattern connecting the x - and y -values of the data points.

Negative correlation



As the x -value increases, the y -value decreases.

No correlation



2. Say whether the data collected on the pairs of variables below would show positive, negative, or no correlation.

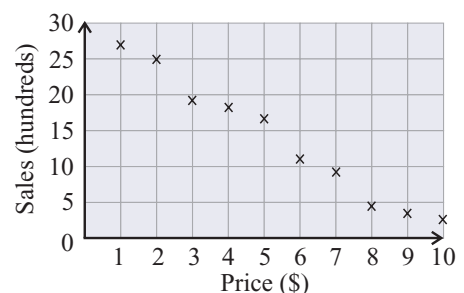
a. The number of pencils you buy, and the total cost of the pencils.

b. The number rolled on a dice, and the outside temperature that day.

c. The number of cars on a freeway, and the speed the cars travel at.

Example

The scatterplot on the right shows the number of sales of a product in one store plotted against various prices the store sold the product for. Describe the correlation of the data. Does this mean that the number of sales of the product increases or decreases as the price is increased?

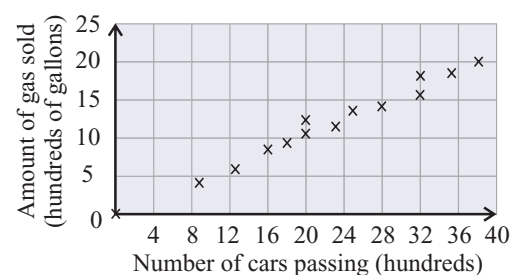


Solution

The graph shows strong negative correlation.

This means that the number of sales decreases as the price increases.

3. The scatterplot on the right shows the number of cars using the road that goes past a gas station in a day, and the amount of gas sold by the gas station that day. How does the amount of gas sold by the station vary with the number of cars using the road?
-
-



4. The data in the table below shows the number of students enrolled in the sixth grade at two schools in neighboring towns over a nine-year period.

School A	160	165	170	180	180	185	200	215	220
School B	105	125	130	135	130	145	160	160	180

- a. Make a scatterplot to compare the two sets of enrollment data on the grid provided.
- b. What kind of correlation does the data display?
.....
- c. Complete this statement: “As the number of sixth graders at School A increased, the number of sixth graders at School B”



5. a. Complete the table below showing the number of inches in certain numbers of feet.

Feet	-----	1	3	4	7	-----
Inches	0	12	-----	48	-----	96

- b. Make a scatterplot of the data on the grid provided.
- c. What kind of correlation does the data display?
.....



6. The table below shows the number of hours a group of students spent studying in a week and the distance that they live from school.

Distance (miles)	1	2	5	5	7	9	11
Hours spent studying	12	8	14	10	9	12	13

- a. Make a scatterplot of the data on the grid provided.
- b. What kind of correlation does the data display?
.....
- c. What effect does the distance from school a student lives have on the length of time they spend studying?
.....



Lesson 6.2.3

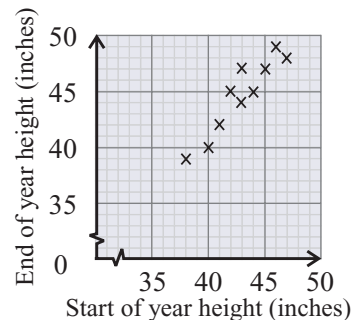
Using Scatterplots

California Standards: Statistics, Data Analysis, and Probability 1.2, Mathematical Reasoning 2.3

Example

Miss Kemble measured the heights of the children in her kindergarten class at the beginning and the end of the school year. The scatterplot on the right shows the measurements that she took.

How tall at the end of the year was the student who was tallest at the beginning of the year? What height is the student who stayed the same height over the course of the school year?



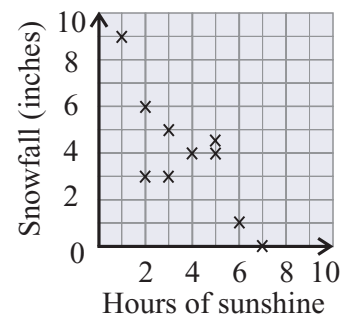
Solution

The student who was tallest at the beginning of the year is represented by the data point that is furthest to the right. Looking at the plot, you can see that the y -value of this coordinate is 48 — so at the end of the year this student was **48 inches** tall.

The point representing the student that stayed the same height over the year must have the same x - and y -values. The only point that this is true for is (40, 40). So the student must be **40 inches** tall.

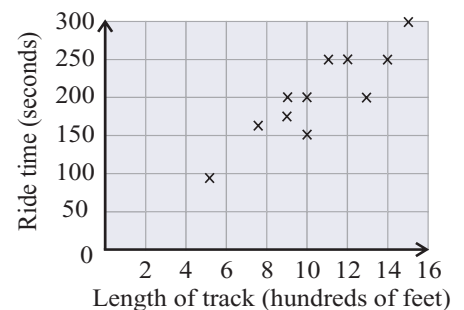
- The scatterplot on the right shows the amount of snow that fell at a California weather station compared to the number of hours of sunshine each day, over the course of 10 days.

- Circle the point representing the day when there was 3 hours of sunshine and 5 inches of snow.
- How much snow fell on the day that there was 6 hours of sunshine?

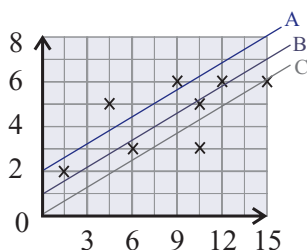


- The scatterplot on the right shows the track lengths of roller coasters compared to the total time that it takes to ride them.

- What is the ride time of the 1300-foot roller coaster?
- Give the lengths of all the roller coasters that take 250 seconds to ride.



3.

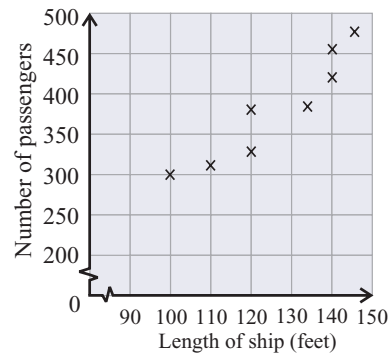


Look at the scatterplot on the left. Explain which line is the line of best fit for this data.

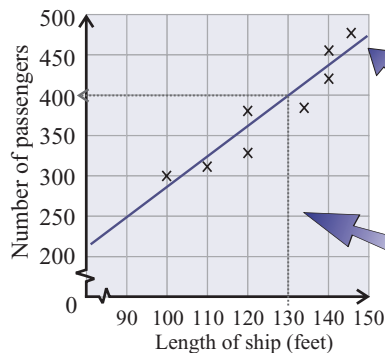
Example

The scatterplot on the right shows the lengths of ten cruise ships plotted against the number of passengers they carry.

Draw a line of best fit on the scatterplot. Use your line to predict roughly how many passengers a 130-foot cruise ship could carry.



Solution



This is a good line of best fit because it splits the data in half, and is close to all the points. There are four points slightly above the line, and four points slightly below the line.

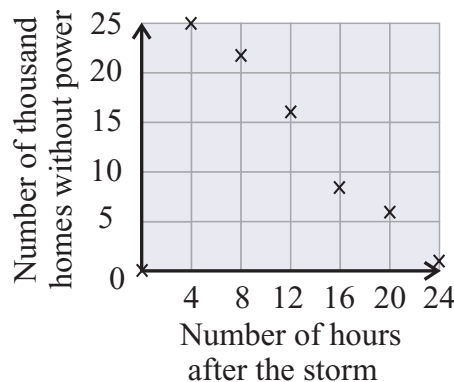
To predict the number of passengers that a 130-foot ship could carry, start at 130 on the x -axis. Draw a line up until you reach the line of best fit. Then draw a line across from this point to the y -axis, and read off the value.

A 130-foot ship could carry approximately **400 passengers**.

4. After a storm a number of homes in a district were left without power. The scatterplot on the right shows the number of houses left without power for the 24 hours after the storm.

- Draw a line of best fit on the scatterplot.
- Describe the correlation of the data.

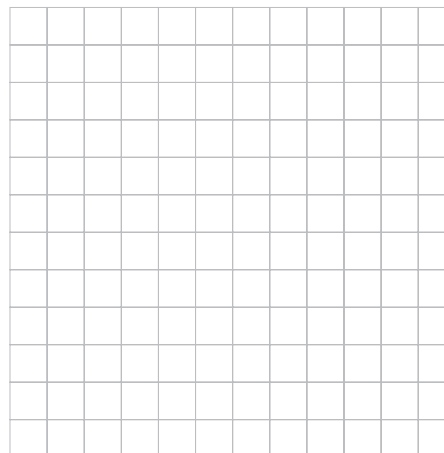
- Roughly how long did it take for power to be restored to all but 17,000 homes?



5. The table below shows the average time that it takes to load an airplane compared to the number of passengers it carries.

Number of passengers	50	115	130	185	200	215	290
Loading time (minutes)	10	12	14	16	20	26	30

- Draw a scatterplot of the data on the grid provided.
- Draw a line of best fit on your plot.
- Use your line of best fit to predict how long it would take to load an airplane carrying 250 passengers.



Lesson
7.1.1

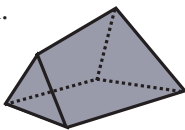
Three-Dimensional Figures

California Standards: Measurement and Geometry 2.1, 3.6

Example

Which of the solids pictured below is not a prism? Explain your answer.

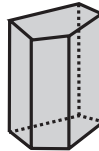
A.



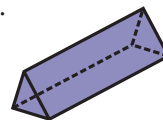
B.



C.



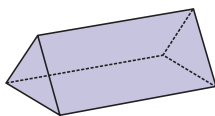
D.



Solution

A is not a prism — the triangle at one end is larger than the triangle at the other, so it doesn't have a constant cross-section.

1.



This solid is a triangular prism.

a. How many faces does it have?

.....

b. How many edges does it have?

.....

c. How many vertices does it have?

.....

2. Say whether each of the statements below is true or false.

a. All prisms have two bases.

.....

b. A pentagonal prism has 5 faces.

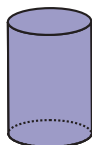
.....

c. A cone has one base.

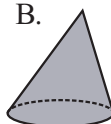
.....

3. The diagrams below are all pictures of three-dimensional solids.

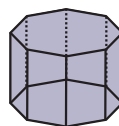
A.



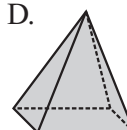
B.



C.



D.



a. Which solid is a cylinder?

.....

b. Which solid is a pyramid?

.....

c. Which solid is a cone?

.....

d. What kind of shape is the other solid? Be as specific as possible.

.....

Example

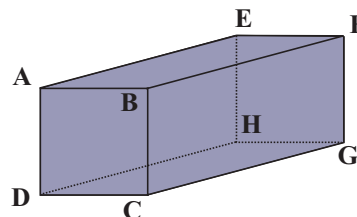
Look at the rectangular prism on the right.
How many diagonals does it have? Name them all by giving their starting vertex and ending vertex.

Solution

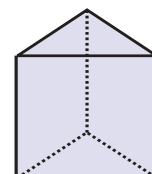
The shape has four possible diagonals.
They are AG, BH, CE, and DF.



A diagonal is a line joining two vertices that aren't on the same face.



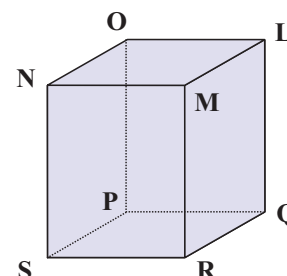
4. The shape on the right is a triangular prism.
How many diagonals does it have?



5. The shape on the right is a rectangular prism.

a. What parts of the prism are LMNO and PQRS?

b. How many diagonals does it have? Name them all by giving their starting vertex and ending vertex.
.....

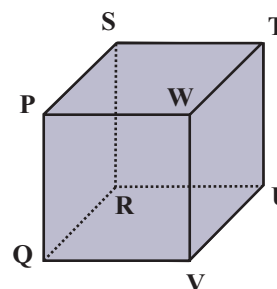


6. Look at the rectangular prism on the right.

a. Which diagonal of the prism starts at vertex T?
.....

b. Which diagonal of the square TUVW starts at vertex T?
.....

c. Are your answers to parts a. and b. the same? Explain.
.....
.....



7. Consider the figure on the right.

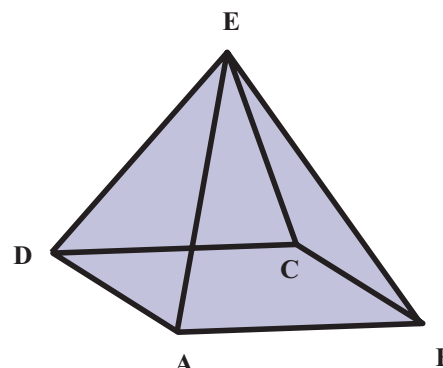
a. What kind of solid is it? Be as specific as possible.
.....

b. How many faces does it have?
.....

c. What shape is the face ABCD?

d. What shape is the face ABE?

e. How many diagonals does it have?
.....



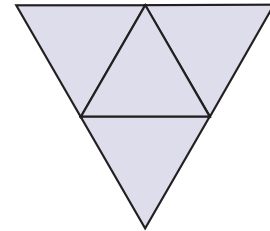
Lesson 7.1.2

Nets

California Standards: Measurement and Geometry 3.5, Mathematical Reasoning 2.2

Example

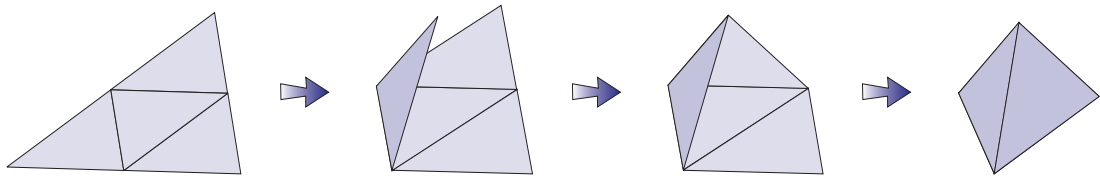
Which three-dimensional shape is this the net of?



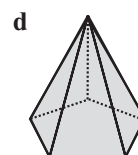
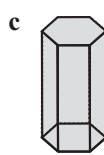
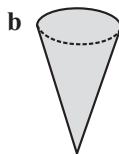
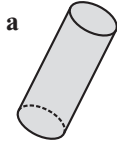
Solution

If you fold along all of the marked lines then you get a pyramid with a triangular base.

So this is the net of a **triangular pyramid**.



1. Match the figures to the nets that they were made from.

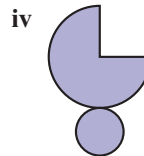
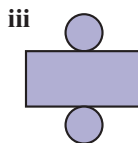
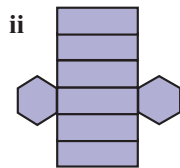
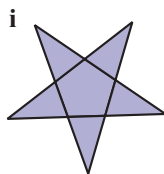


a is made from net

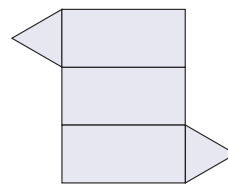
b is made from net

c is made from net

d is made from net



2. What three-dimensional shape is this the net of?

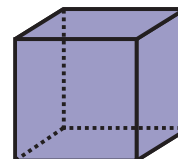


.....

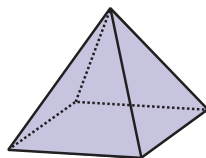
3.



Draw the net of the solid shown below in the box on the left.

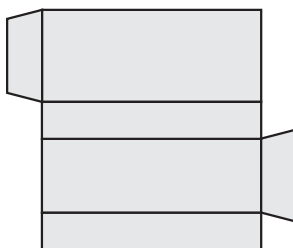


4. a. What kind of solid is shown on the right?



- b. Draw the net of the shape in the box on the right.

5. In the box on the right, draw the three-dimensional shape that would result from folding up this net.



6. In the box on the right, draw a diagram showing the net of a cone that has a slant height of 4 cm and a base radius of 3 cm.



7. In box a., draw a triangular prism. In box b., draw a net that could be used to make your prism.



8. Sandie is making a cone. She wants it to have a height of 40 cm and a base diameter of 60 cm. Draw a diagram showing a net that Sandie could use to make the cone, marking on any measurements you think are necessary. Use the box on the right.



Lesson
7.1.3

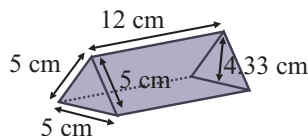
Surface Areas of Cylinders and Prisms

California Standards: Measurement and Geometry 2.1, 3.5, Mathematical Reasoning 1.3

1. What is the general formula for the surface area of a cylinder or prism?
-

Example

What is the surface area of this prism?



Solution

Draw the net of the prism.

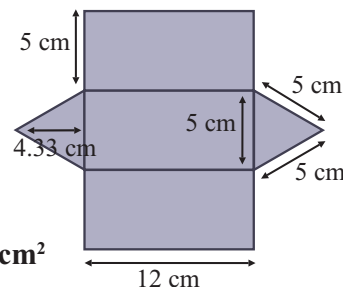
The net of the prism has three identical rectangles.

The area of each rectangle is $12 \text{ cm} \times 5 \text{ cm} = 60 \text{ cm}^2$.

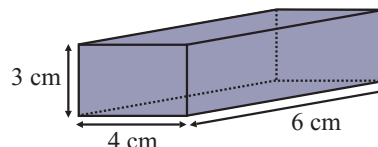
It also has two identical triangles.

The area of each triangle is $\frac{1}{2} \times 5 \text{ cm} \times 4.33 \text{ cm} \approx 10.825 \text{ cm}^2$

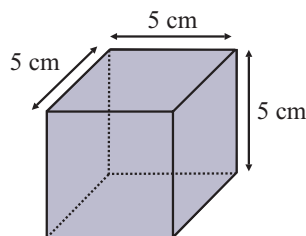
So the total surface area of the prism $\approx (3 \times 60) + (2 \times 10.825) = 201.65 \text{ cm}^2$



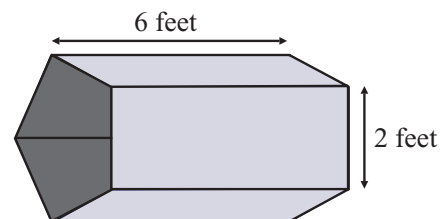
2. What is the surface area of this prism?
-



3. What is the surface area of this prism?
-

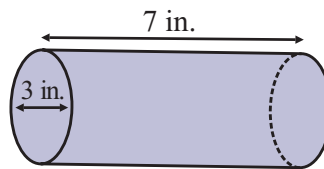


4. The figure on the right shows a hollow tube that Lorcan uses as part of his conjuring act. He wants to paint the outside of the tube red, and needs to know how much paint to buy. What is the area of the outside of the tube?
-



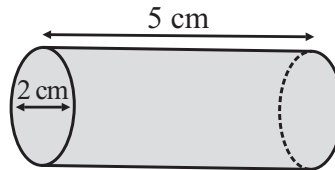
5. What is the surface area of this cylinder?
Leave your answer in terms of π .

.....



6. What is the surface area of this cylinder?
Round your answer to one decimal place.

.....



7. Which of the two figures shown below has the greater surface area?

Figure A:

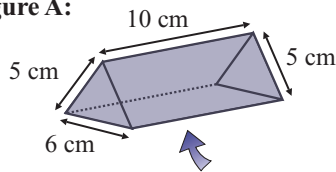
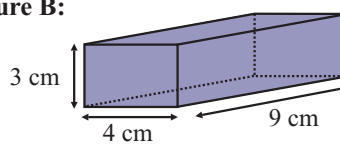


Figure B:

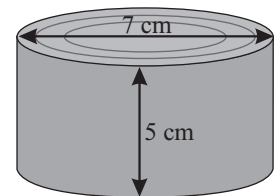


This time the rectangles that make up the triangular prism's sides aren't all the same size

.....

8. A company is making labels to cover the lateral area of tins of the dimensions shown in the diagram. What total area of paper, in cm^2 , will be needed to make labels for 200 tins? Use $\pi = 3.14$.

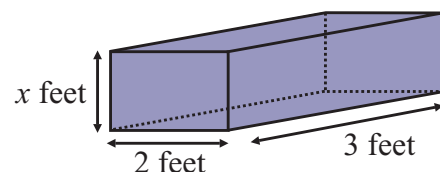
.....



9. The rectangular prism on the right has a length of 3 feet, a width of 2 feet, and a height of x feet.

- a. Complete the table below to show the surface area of the prism when x is equal to the values in the first column.

x	Surface area (ft^2)
1 foot
2 feet
3 feet
4 feet



- b. Use the pattern in the table to predict what the surface area of the prism will be when $x = 6$ feet.

.....

Lesson
7.1.4

Surface Areas & Perimeters of Complex Shapes

California Standards: Measurement and Geometry 2.1, 2.2, 2.3, Mathematical Reasoning 1.3

Example

Find the total edge length of the complex shape shown on the right.

Solution

Splitting the shape into a rectangular prism and a cube:

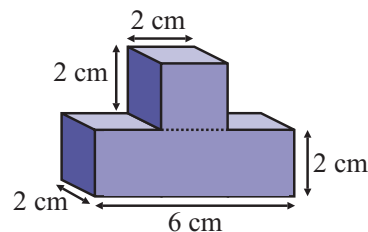
The cube has an edge length of $12 \times 2 = 24$ cm

The prism has an edge length of $(2 \times 8) + (6 \times 4) = 40$ cm

But two of the cube's edges aren't edges of the combined shape. So you need to subtract these shared edges from the edge length of the cube, and from the edge length of the prism.

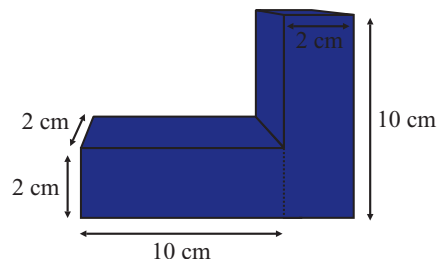
So the total edge length of the shape is:

$$24 + 40 - 2(2 \times 2) = 64 - 8 = \mathbf{56 \text{ cm}}$$



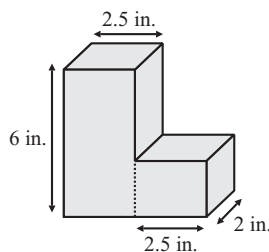
In Exercises 1–4 find the total edge length of the complex shapes.

1.



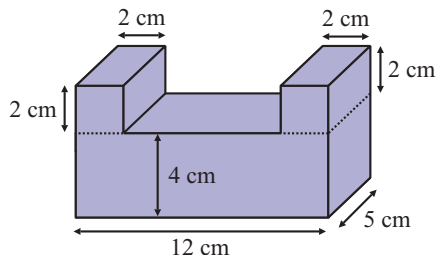
Edge length =

2. Edge length =

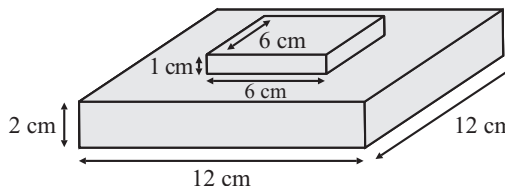


Edge length =

3.



4. Edge length =



Example

Find the total surface area of the complex shape shown on the right.

Solution

Split the shape into a rectangular prism and a cube:

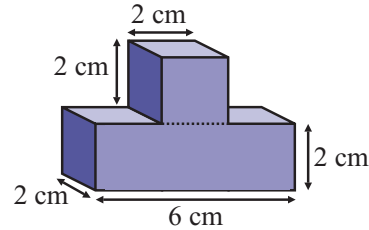
The cube has an area of $6 \times 2 \times 2 = 24 \text{ cm}^2$

The prism has an area of $(4 \times 6 \times 2) + (2 \times 2 \times 2) = 56 \text{ cm}^2$

But one of the cube's faces isn't a face of the combined shape. So you need to subtract this shared face from the surface area of the cube, and from the surface area of the prism.

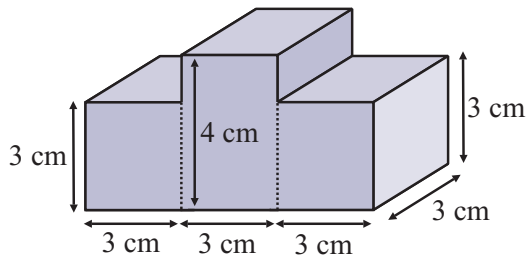
So the total surface area of the shape is:

$$24 + 56 - 2(2 \times 2) = 80 - 8 = \mathbf{72 \text{ cm}^2}$$



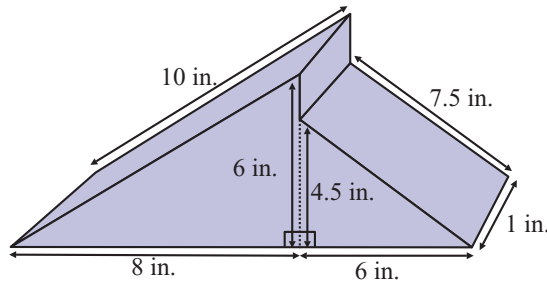
In Exercises 5–8 find the total surface area of the complex shapes. Use $\pi = 3.14$.

5.

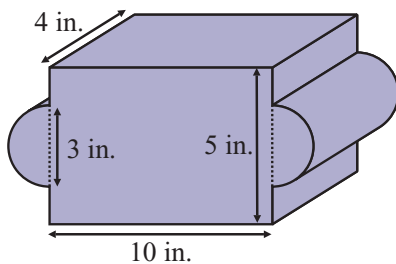


Surface area =

6. Surface area =



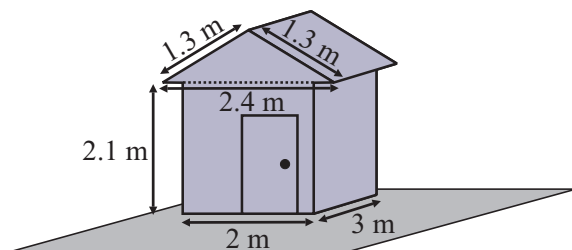
7.



Surface area =

8. Fabio is going to paint his garden shed. A diagram of the shed is shown on the right. A can of paint covers about 6 m^2 . Given that he wants to paint all the outside walls, the door, and the roof, how many cans should Fabio buy?

.....



Lesson
7.1.5

Lines and Planes in Space

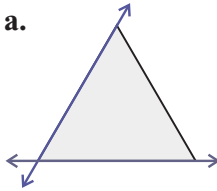
California Standards: Measurement and Geometry 3.6

1. Line A crosses line B at point x . Are lines A and B coplanar or skew?
-

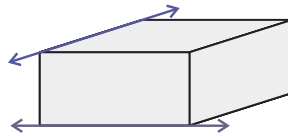
Example

Say whether the following pairs of lines are coplanar or skew.

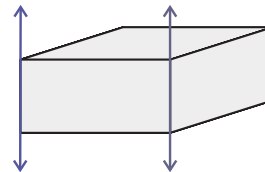
a.



b.



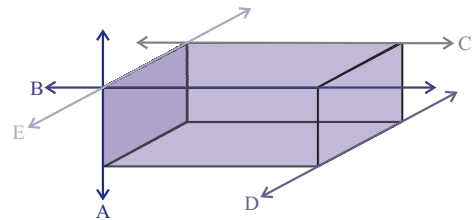
c.



Solution

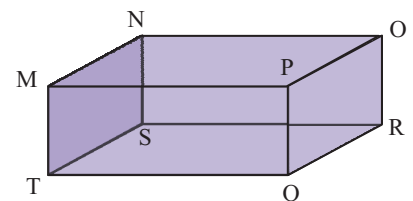
- a. These lines intersect. That means they are coplanar. They both rest on the plane of the triangle.
 b. These lines don't intersect, and aren't parallel. That means they aren't coplanar. So they are skew.
 c. These lines are parallel. That means they are coplanar. They both rest on the plane of the front face of the rectangular prism.

Exercises 2–5 relate to the diagram on the right.



2. Give the letters of any lines that are coplanar with line B. -----
3. Give the letters of any lines that are skew to line B. -----
4. Give the letters of any lines that are coplanar with line D. -----
5. Give the letters of any lines that are skew to line A. -----
6. Use the rectangular prism below to say whether the following statements are true or false.

- a. MN and OP are parallel. -----
- b. SN and ST are skew. -----
- c. ST and OP are parallel. -----



7. Look at the diagrams below. Say which diagram is described by statements a, b, and c.

DIAGRAM 1:

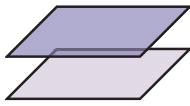


DIAGRAM 2:

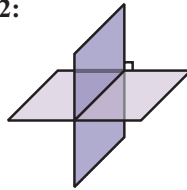
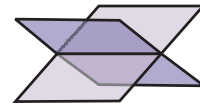


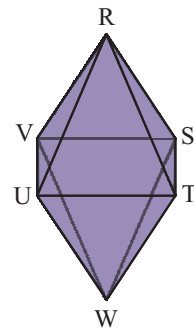
DIAGRAM 3:



- a. The blue plane intersects perpendicularly with the gray plane.
- b. The blue plane is parallel to the gray plane, and they never intersect.
- c. The blue plane intersects with the gray plane, but not perpendicularly.

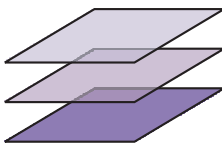
8. Look at the diagram on the right.

- a. Which line segments are parallel to ST?
- b. Which line segments are skew to WT?
- c. Which line segments are coplanar with SV?



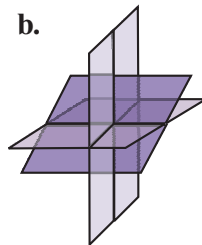
9. Say whether the planes in the diagrams below meet, and if so, how.

a.



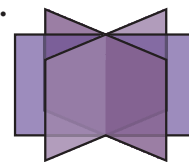
.....

b.



.....

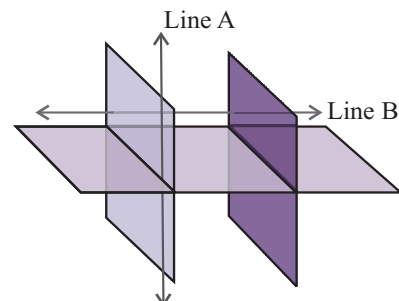
c.



.....

10. The diagram on the right shows a group of planes and lines.

- a. Are lines A and B coplanar or skew?
- b. Say how line A relates to the light blue plane.
- c. Say how line B relates to the light blue plane.
- d. Say how the light blue plane relates to the dark blue plane.
- e. Say how the dark blue plane relates to the gray plane.



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Lesson
7.2.1

Volumes

California Standards: Measurement and Geometry 2.1, Mathematical Reasoning 2.2, 3.2

Example

Find the volume of the solid shown on the right.

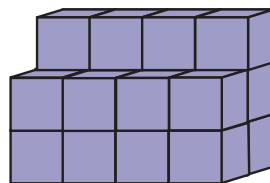
Solution

Count the number of cubes that make up the shape.

There are eight on the bottom layer, eight on the middle layer, and four on the top layer. So there are $8 + 8 + 4 = 20$ cubes making up the solid.

Each small cube represents 1 cubic unit.

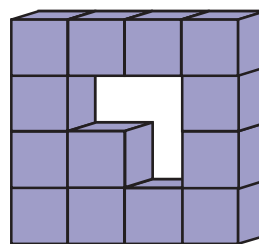
So the volume of the solid is **20 cubic units**.



Each cube is one unit high, one unit wide, and one unit deep. So the volume of each cube is 1 cubic unit.

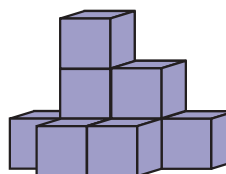
1. Find the volume of the solid shown on the right.

.....

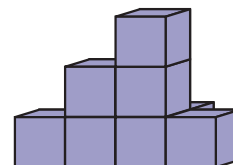


2. Find the volume of the solid shown on the right.

.....



Front



Back

3. What is the general formula for finding the volume of a prism or cylinder?

.....

Example

Find the volume of the prism shown on the right.

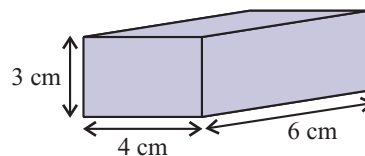
Solution

The base of this prism is a rectangle. First find its area.

Area of the rectangle = length \times width = $4 \text{ cm} \times 3 \text{ cm} = 12 \text{ cm}^2$

Now multiply the area of the base by the height of the prism.

Prism volume = base area \times height = $12 \text{ cm}^2 \times 6 \text{ cm} = 72 \text{ cm}^3$

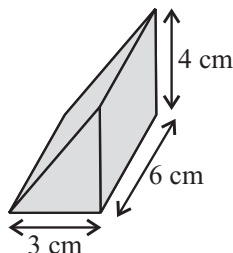


4. If you find the volume of a cube with an edge length of 4 feet, what will the units of your answer be?

.....

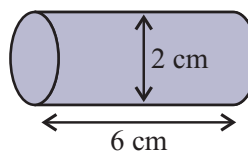
5. Find the volume of the prism shown on the right.

.....



6. Find the volume of the cylinder shown on the right.
Give your answer to one decimal place.

.....

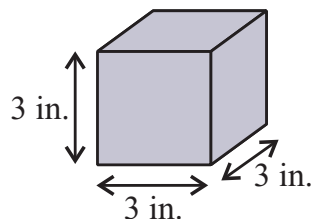


7. a. What formula can you use to find the volume of a cube?

.....

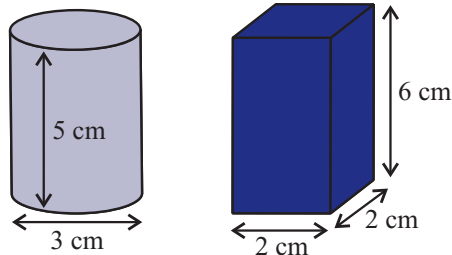
- b. Find the volume of the cube shown on the right.

.....



8. Which of the two solids pictured on the right has the bigger volume?

.....



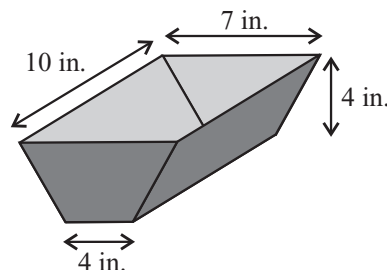
9. Shona is baking a cake. Her recipe says that she should use a circular cake tin with a diameter of 8 inches and a depth of 4 inches. The only tin Shona has is the one shown on the right.

- a. Find the volume of a cylinder with a base diameter of 8 inches, and a height of 4 inches. Give your answer to 1 decimal place.

.....

- b. Does Shona's tin have a large enough volume to hold the amount of mixture that the correct round tin would have held?

.....



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Lesson 7.2.2

Graphing Volumes

California Standards: Algebra and Functions 3.2, Mathematical Reasoning 2.3

Example

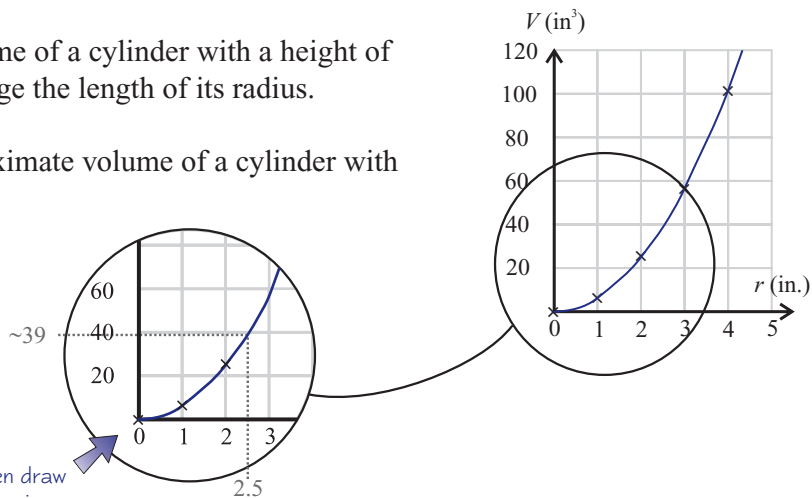
This graph shows how the volume of a cylinder with a height of two inches changes as you change the length of its radius.

Use the graph to find the approximate volume of a cylinder with a radius of 2.5 inches.

Solution

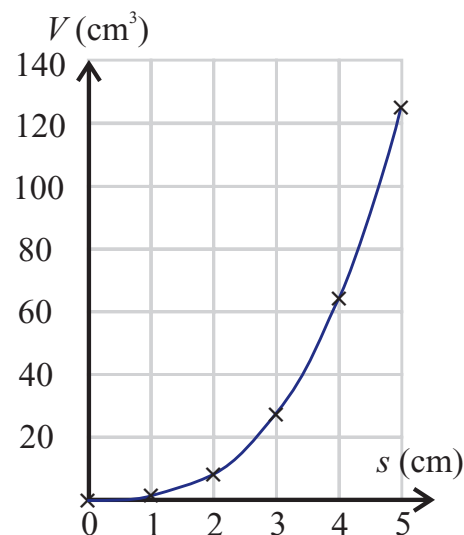
Approximately **39 inches³**.

Draw a line up from 2.5 on the x -axis, then draw across and read off the value on the y -axis.

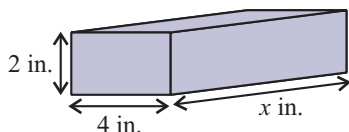


1. The graph on the right shows how the volume of a cube changes with its side length, s . Use the graph to answer the questions below.

- Find the approximate volume of a cube with a side length of 3.5 cm.
- Find the approximate volume of a cube with a side length of 4.4 cm.
- Find the approximate side length of a cube with a volume of 10 cm³.



2. The questions below are about this rectangular prism:



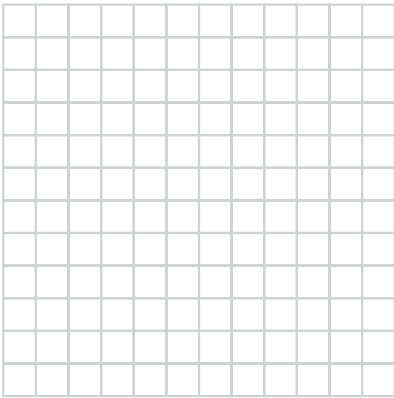
x = prism height (in.)

- Write an expression, using the variable x , that you could use to find the volume of this prism.
- Complete the table on the right showing how the volume of this prism changes with its height, x .

Prism height (x in.)	Prism volume (in ³)
.....	0
1
2	16
.....	24
4
5

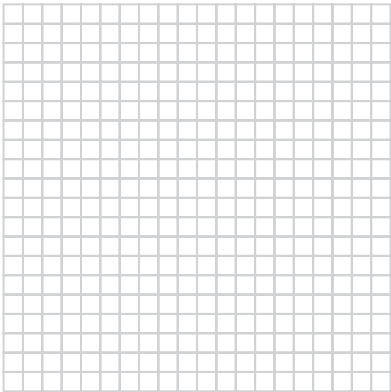
3. The base of a rectangular prism has an area of 4 cm^2 .
- Complete the table below showing how the volume of the prism changes with its height.
 - Graph the volume of the prism against its height, using the grid below.
 - Use your graph to estimate the volume of the prism if it has a height of 4.5 cm.
 - Use your graph to estimate the height of the prism if it has a volume of 10 cm.

Prism height (cm)	Prism volume (cm^3)
0
1	4
2
.....	12
4
5

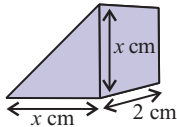


4. This question is about a cylinder with a height of 10 cm.
- Complete the table below showing how the volume of the cylinder changes with its base radius.
 - Graph the volume of the cylinder against its base radius. Use the grid below.
 - Use your graph to estimate the volume of a cylinder with a base radius of 1.5 cm.

Base radius (cm)	Cylinder volume (cm^3)
0	0
1
2	125.7
3
4



5. This question is about the volume of this triangular prism:



- Draw a graph to show how the volume of the prism changes as x changes from 0 to 10. Use the grid on the right.
- Use your graph to find the volume of the prism when $x = 5.5$.
.....
- Use your graph to estimate the value of x when the volume of the prism is 72.25 cm^3



Lesson
7.3.1

Similar Solids

California Standards: Measurement and Geometry 1.2

1. If a shape is multiplied by a scale factor that is between 0 and 1, what will happen to its side lengths?
-

2. Triangle A is multiplied by a scale factor of 2.5 to produce Triangle B.

a. Will the corresponding angles in triangles A and B be of equal measure?

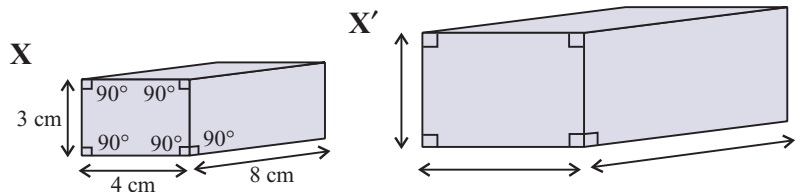
b. Will the corresponding side lengths in triangles A and B be the same?

Example

X and X' are similar solids.

X has been multiplied by a scale factor of 1.5 to get X'.

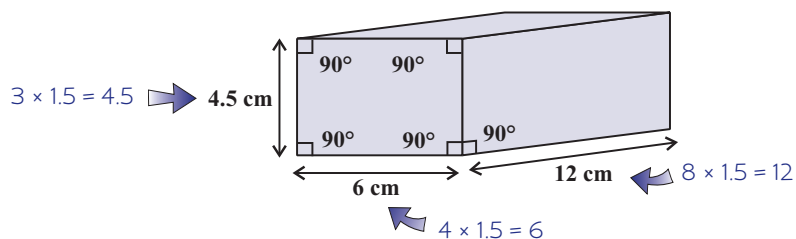
Label the missing side lengths and angles in X'.



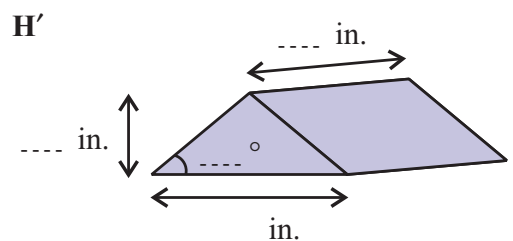
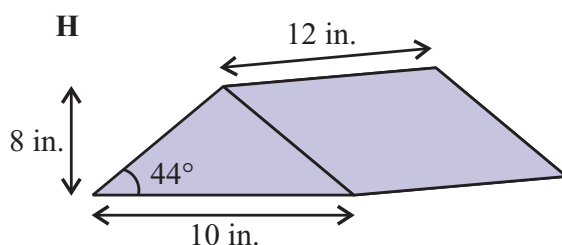
Solution

Since the solids are similar, all the angles in X and X' must be of equal measure.

X has been multiplied by 1.5 to produce X'. So to find the side lengths in X' you need to multiply the corresponding side lengths in X by 1.5.

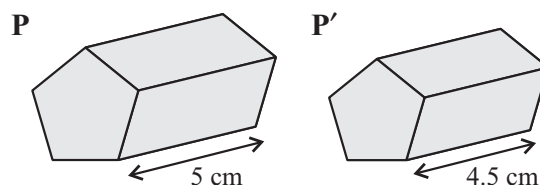


3. H and H' are similar solids. H has been multiplied by a scale factor of $\frac{3}{4}$ to produce H'. Label H' with the missing side lengths and angles indicated.



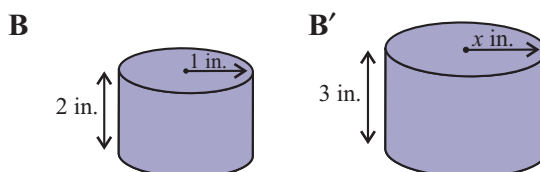
4. The solids P and P' are similar. What scale factor has P been multiplied by to produce P'?

.....



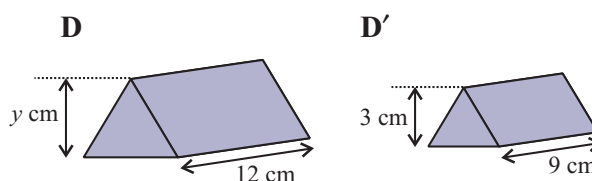
5. The solids B and B', shown on the right, are similar. Find the missing length, x in.

.....



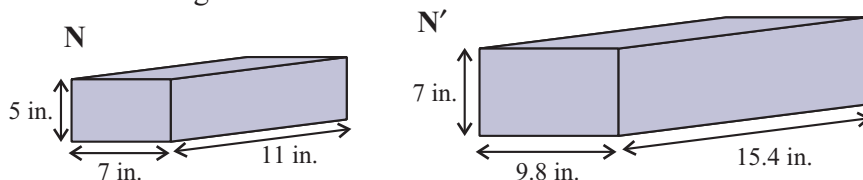
6. The solids D and D', shown on the right, are similar. Find the missing length, y cm.

.....

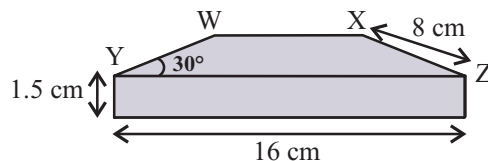


7. Are the two rectangular prisms shown on the right similar?

.....

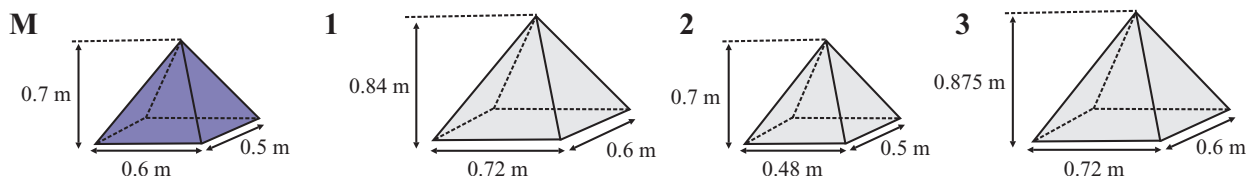


8. A shop wants a new window display platform. The shop staff make a model of their chosen design. A diagram of the model is shown on the right. The scale factor from the model to the real platform will be 12.



- a. What will the height of the real platform be?
- b. What will the length of side XZ be in the real platform?
- c. What will the measure of angle WYZ be in the real platform?

9. Which of the numbered figures below is similar to Figure M? What scale factor has been used to produce it?



Lesson
7.3.2

Surface Areas & Volumes of Similar Figures

California Standards: Measurement and Geometry 1.2, 2.1, 2.3

- The surface area of a solid is $x \text{ cm}^2$ and its volume is $y \text{ cm}^3$. The solid is multiplied by a scale factor of 2 to create a new solid.
 - What is the surface area of the new solid?
 - What is the volume of the new solid?

Example

K and K' are similar solids. What scale factor has K been multiplied by to produce K'? Use the scale factor to find the surface area and volume of K'.

Solution

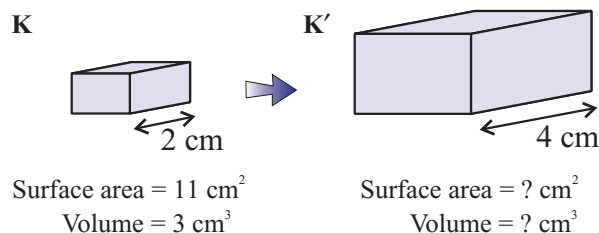
The length of 4 cm in solid K' corresponds to the length of 2 cm in solid K.

Scale factor = length in K' \div length in K = $4 \div 2 = 2$

This means that the side lengths in K' are two times greater than the side lengths in K. So the surface area of K' must be $2 \times 2 = 4$ times greater than that of K, and the volume $2 \times 2 \times 2 = 8$ times greater.

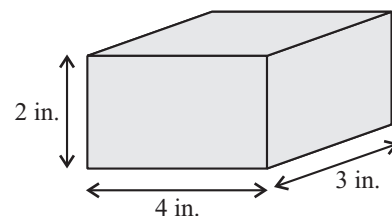
Surface area of K' = $11 \times 4 = 44 \text{ cm}^2$

Volume of K' = $3 \times 8 = 24 \text{ cm}^3$



- What is the surface area of the prism shown on the right?
.....

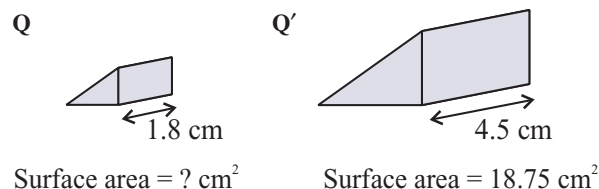
- If the prism is multiplied by a scale factor of four, what will its new surface area be?



- Q and Q' are similar solids.

- What scale factor has Q been multiplied by to produce Q'?

- Find the surface area of Q.



- A rectangular prism is multiplied by a scale factor of x to produce a new solid. The surface area of the original solid was 22 in^2 , and the surface area of the new solid is 198 in^2 . What is the value of x ?
.....

5. The diagram on the right shows the nets of two similar solids.

a. What is the area of net 1?

.....

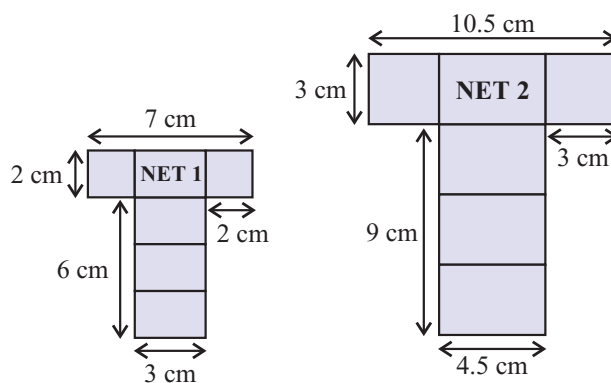
b. What is the area of net 2?

.....

c. Both nets are folded into rectangular prisms.

Use the areas that you calculated in parts a. and b.

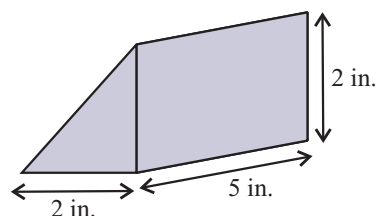
to find what scale factor you would have to multiply the prism made from net 1 by to get the prism made from net 2.



6. a. What is the volume of the prism shown on the right?

.....

b. If the prism is multiplied by a scale factor of three, what will the volume of the image be?



7. The two cylinders shown on the right are similar solids.

a. Find the surface area of cylinder 1. Leave your answer in terms of π .

.....

b. Find the surface area of cylinder 2. Leave your answer in terms of π .

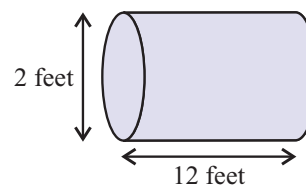
.....

c. Use the areas you have calculated to prove that cylinder 1 has been

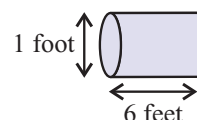
multiplied by a scale factor of $\frac{1}{2}$ to produce cylinder 2.

.....

Cylinder 1



Cylinder 2



8. a. Find the volume of the prism shown on the right.

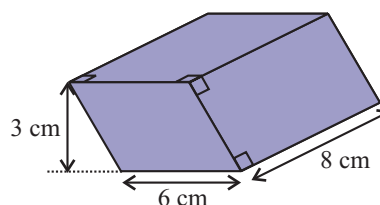
.....

b. The prism is multiplied by a scale factor of 1.4. What will the volume of the image be?

.....

c. The original prism is multiplied by a scale factor of $\frac{x}{4}$. x is an integer. The volume of the new prism is 60.75 cm^3 . What is the value of x ?

.....



Lesson
7.3.3

Changing Units

California Standards: Measurement and Geometry 2.4

1.
 - a. How many centimeters are in one meter?
 - b. How many inches are in one foot?
 - c. How many centimeters are in one inch?
 - d. Approximately how many meters are in one foot?
2. Give your answers to the following to one decimal place.
 - a. Find the area, in cm^2 , of a 16 inch by 14 inch rectangle.
 - b. Find the surface area, in m^2 , of a cube with a 1 foot side length.
 - c. Find the volume, in cm^3 , of a cube with a 4 inch side length.
 - d. Find the volume, in m^3 , of a cylinder with a 4 foot radius and a 16 foot height.

Example

A rectangular prism has a volume of 2 feet^3 . What is the volume of the prism in cubic inches?

Solution

First you need to work out how many cubic inches are contained in one cubic foot:

$$1 \text{ foot}^3 = 12 \text{ in.} \times 12 \text{ in.} \times 12 \text{ in.} = 1728 \text{ in}^3$$

This is the conversion factor.

Use this to write a proportion:

$$\frac{1728}{1} = \frac{x}{2}$$

This variable represents the volume of the prism in inches.

This is the prism's volume in feet^3 .

Cross-multiply, and solve for x :

$$1 \times x = 1728 \times 2$$

$$x = 3456$$

So the volume of the prism is **3456 in^3**

3. A prism has a surface area of 1 m^2 . What is the surface area of the prism in cm^2 ?
4. A cube has a volume of 500,000 cm^3 . What is the volume of the cube in m^3 ?
5. A prism has a surface area of 36 in^2 . What is the surface area of the prism in feet^2 ?

6. A cube has a volume of 5 feet³. What is the volume of the cube in in³?

.....

7. A prism has a surface area of 200 in². What is the surface area of the prism in cm²?

.....

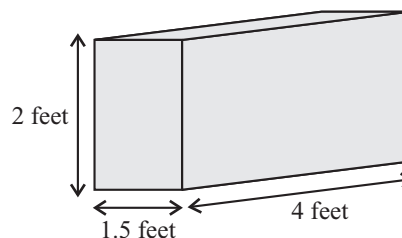
8. This question is about the prism shown on the right.

- a. Find the surface area of the prism in feet².

.....

- b. Find the surface area of the prism in in².

.....



9. This question is about the cylinder shown on the right.

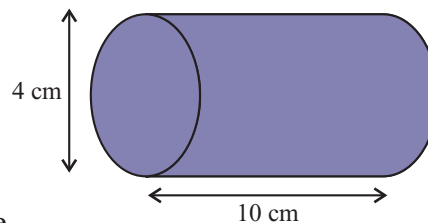
- a. Find the volume of the cylinder in cm³.

Leave your answer in terms of π .

.....

- b. Convert this volume to in³. Give your answer to one decimal place.

.....



10. F and F', shown on the right, are similar solids.

- a. What is the surface area of F in m²?

.....

- b. What scale factor has been applied to F to produce F'?

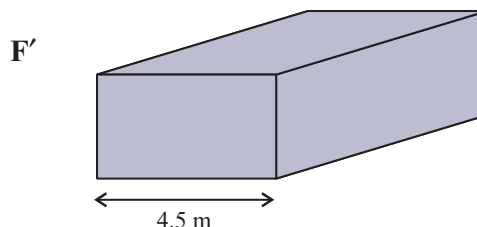
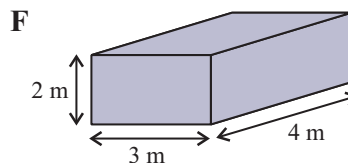
.....

- c. What is the surface area of the enlarged solid in cm²?

.....

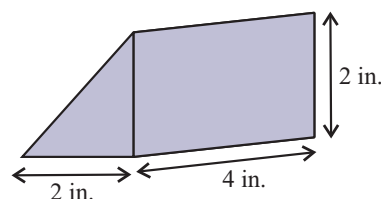
- d. What is the surface area of the enlarged solid in feet²?

.....



11. The prism shown on the right is enlarged by a scale factor of 2.1. What is the volume of the enlarged solid in cm³? Give your answer to one decimal place.

.....



Lesson
8.1.1

Percents

California Standards: Number Sense 1.3

Example

Write the fraction $\frac{34}{100}$ as a decimal and a percent.

Solution

$$\frac{34}{100} = 0.34, \quad \frac{34}{100} = 34\% \quad \leftarrow \text{A percent tells you how many hundredths you have.}$$

1. Write the following fractions as percents.

a. $\frac{7}{100}$ b. $\frac{18}{100}$ c. $\frac{100}{100}$ d. $\frac{86}{100}$

2. Write each of the following percents as a fraction in its simplest form.

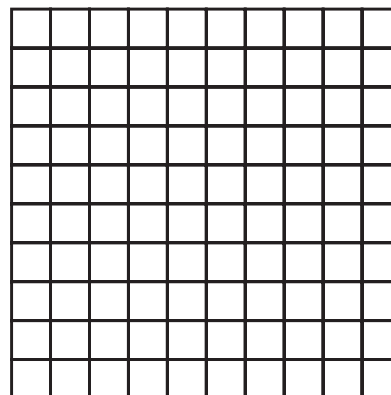
a. 50% b. 20% c. 3% d. 12%

3. Nick shared an apple pie with his family.
Altogether they ate 78% of the pie.

a. Shade a region of the diagram on the right to show how much of the pie was eaten.

b. What percent of the pie was left?

.....



4. Write each of the following as a percentage.

a. $\frac{155}{100}$ b. $\frac{400}{100}$ c. $\frac{2600}{100}$ d. $\frac{267}{100}$

e. 1 f. 1.8 g. 2.4 h. 54

5. Find:

- a. 25% of 80 _____
- b. 4% of 150 _____
- c. 70% of 560 _____
- d. 56% of 800 _____
- e. 2% of 740 _____
- f. 6.5% of 44 _____

6. 10% of Alejandro's class said that red was their favorite color, 30% said that blue was their favorite color, and 20% said that green was their favorite color. If there are 30 people in his class, how many students said that their favorite color was:

- a. red? _____
- b. blue? _____
- c. green? _____

7. Puebla's baseball team has won 8 games, 40% of their games this season. Write and solve an equation to find how many games the team has played. Show your work.

8. In a survey, 36% of the people asked said they find out the latest news from their radio. If 324 people gave this answer, how many people were surveyed?

9. Jennifer does a survey of her school. She finds that 18% of the 250 seventh graders keep a diary, compared to only 16% of the 200 eighth graders.

- a. How many more seventh graders than eighth graders keep a diary?

- b. What total percentage of all seventh and eighth graders keep a diary?

Lesson
8.1.2

Changing Fractions and Decimals to Percents

California Standards: Number Sense 1.3

Example

Write each of the following as a percent.

a. 0.8 b. $\frac{1}{5}$

Solution

a. $0.8 \times 100 = 80\%$

b. Let n = the number you need to multiply 5 by to get 100.
Then $5n = 100$, and $n = 100 \div 5 = 20$.

$\frac{1}{5} = \frac{1 \times 20}{5 \times 20} = \frac{20}{100} = 20\%$

Multiply decimals by 100 to get a percent.

Multiply both the numerator and the denominator by n to get an equivalent fraction that can easily be written as a percent.

1. Write each of the following decimals as a percent.

a. 0.5 _____ b. 0.9 _____ c. 0.25 _____ d. 0.1 _____

2. Write each of the following decimals as a percent.

a. 0.87 _____ b. 0.03 _____ c. 0.17 _____ d. 0.51 _____
e. 1.45 _____ f. 5.01 _____ g. 7.68 _____ h. 87.1 _____

3. Write each of the following fractions as a percent.

a. $\frac{29}{100}$ _____ b. $\frac{1}{2}$ _____ c. $\frac{5}{50}$ _____ d. $\frac{1}{4}$ _____

4. Write each of the following fractions as a percent.

a. $\frac{7}{20}$ _____ b. $\frac{5}{8}$ _____ c. $\frac{5}{1000}$ _____ d. $\frac{130}{650}$ _____

5. A quarterback completed 9 out of 15 passes during a football game.
- What fraction of the passes did the quarterback complete?
 - What percent of the passes did the quarterback complete?
6. In a survey about young people's spending habits, 69 out of the 120 teenagers questioned said they get a regular weekly allowance.
- What percent of the teenagers questioned get a regular weekly allowance?
Round your answer to the nearest percent.
.....
 - Out of the those who get a regular weekly allowance, 23 said that they also have a weekend job. What percent of the teenagers questioned who receive a regular weekly allowance do not have a weekend job? Round your answer to the nearest percent.
.....
7. A sports store sold \$171,000 of fishing gear and \$284,400 of golf clubs last year. If their total sales for last year were \$900,000, what percentage of their sales were from:
- fishing gear?
 - golf clubs?
8. Isabel has spent a total \$1850 on computer equipment for her home. The equipment included a mouse for \$12.95, a computer monitor for \$323.75, and a printer for \$64.75. Find the percent of the total cost of each of the following items. Show your working.
- The monitor.
 - The printer.
 - The mouse.
9. Graciela is saving up \$800 for flights to Europe. She has already saved \$392.
- What percent is this of the total she needs before she can pay for the flights?
.....
 - What percent of the total cost of the flights does she still need to save?
.....

Lesson
8.1.3

Percent Increases and Decreases

California Standards: Number Sense 1.3, 1.6

1. Find the total for each of the following increases. Show your working.

- a. 100 increased by 72%.
- b. 10 increased by 30%.
- c. 80 increased by 60%.
- d. 6 increased by 10%.
- e. 12 increased by 40%.

2. Lorenzo goes out for dinner. His bill comes to \$22. He wants to leave a 15% tip. How much money should he leave in total?
-

3. Find the percent increase in the following cases.

- a. A company increases its number of staff from 90 to 135.
- b. A store increases its opening hours from 7.5 to 10 hours a day.
- c. Ray has 300 football cards, and buys 27 more at the store.
- d. The number of students passing a school science test has increased from 25 to 32.

4. Helga is comparing sales figures for three stores in a mall over two days. Complete the table to show the percent increase in the sales for each store.

Store	Sales for day 1	Sales for day 2	% increase
1	6500	7839	
2	1200	1350	
3	3400	3978	

Example

Heather's website keeps a log of how many times it is visited each day. It was visited 50 times on Monday and 66 times on Tuesday. Find the percent increase in the number of visits from Monday to Tuesday.

Solution

$66 - 50 = 16$. So the percent increase in the number of visits = $\frac{16}{50} \times 100 = 32\%$

5. Find the total after the following decreases.
- 50 decreased by 3%.
 - 82 decreased by 15%.
 - 640 decreased by 24%.
 - A portion of low-fat lasagna contains 25% less fat than the 15 g that the full-fat alternative contains.
.....
6. Find the percent decreases for the following:
- A necklace whose price is dropped from \$40 to \$10.
 - Lemons are discounted from \$1 each to 70 cents.
 - Kylie has 250 marbles but loses 40.
 - Joey gave away 6 out of his 20 baseball cards.
7. A sports store is having a sale. Footballs that originally cost \$22 are now \$16.50, and baseball bats are down from \$40 to \$32.
- Find the percent decrease in the price of a football.
 - Find the percent decrease in the price of a baseball bat.
 - Which offer gives the largest percent decrease in cost?
8. The table on the right shows the number of copies of a best-selling book sold at a bookstore in five weeks.
- What is the percent increase from Week 2 to Week 3?
Round your answer to the nearest tenth.
.....
 - What is the percent decrease from Week 3 to Week 4?
Round your answer to the nearest tenth.
.....
- | Week | No. of books sold |
|--------|-------------------|
| Week 1 | 1250 |
| Week 2 | 1400 |
| Week 3 | 1900 |
| Week 4 | 1800 |
| Week 5 | 2232 |
9. Marcos is writing an article on how prices changed between the years 2000 and 2005. The average daily rate for a hotel chain was \$66 in 2000 and \$82.50 in 2005.
- Find the percent of increase for the average daily rate from 2000 to 2005.
 - In 2005, customers were able to get 10% off the hotel's daily rate by booking online.
Find the daily rate that online customers were charged in 2005.

Lesson
8.2.1

Discounts and Markups

California Standards: Number Sense 1.3, 1.7

1. Find the sale price for each of the following items.

a. A \$48 sweater, discounted by 50%.

b. A \$180 MP3 player, discounted by 30%.

c. A \$62 pair of shoes, discounted by 35%.

d. A \$129 twin size quilt, discounted by 15%.

e. A \$56 watch, discounted by 23%.

2. An athletic store marks up tennis shoes 75% from \$36, and tennis rackets 40% from \$24. Find the retail price of:

a. tennis shoes

b. a tennis racket


Example

Brione buys a pair of gloves in a 20% sale.

She also gets a further 10% off the discounted price with her student discount.

If the retail price of the gloves was \$7.50, how much does Brione pay for the gloves?

Solution

$20\% = \frac{20}{100}$, so discount on gloves = $\frac{20}{100} \times 7.5 = \1.50 ,  Find the discounted price from the sale.

Sale price of gloves = $\$7.5 - \$1.5 = \$6$.

Student discount = $10\% = \frac{10}{100}$, so discount on sale price of gloves = $\frac{10}{100} \times 6 = \0.60  Find the discount on the already discounted price.

Brione pays = $\$6 - \$0.60 = \$5.40$

3. A department store is offering a 25% discount on everything in stock. Jackie decides to buy a sweater that has a retail price of \$34.

a. What is the discounted price of the sweater?

b. When Jackie gets to the cashier, she notices a small mark on the sweater and is given a further 10% off the discounted price.

How much does Jackie pay for the sweater?

4. A shoe store is having a 15% discount sale. Pedro buys a pair of boots on a day when they offer an extra 30% off the discounted price. How much did the pair of boots cost him if their original price was \$80?

5. Two \$8 sweaters are in a sale. One is discounted by 20%, and the other is discounted by 10% and then a further 10% off the discounted price. Which sweater will be cheaper? Show your work.

.....

.....

.....

6. Find the retail price of each item sold at a hardware store if they are sold with the percent markup shown.
- a. A lawn mower: wholesale price \$200, markup 50%.
 - b. A garden hose: wholesale price \$10, markup 80%.
 - c. A park bench: wholesale price \$70, markup 20%.
 - d. A camping stove: wholesale price \$18, mark up 65%.
 - e. A tool kit: wholesale price \$62, mark up 56%.
 - f. A can of paint: wholesale price \$12, mark up 83%.

7. The table on the right shows the percent markup for different items found at a bookstore. Find the retail price of the following items.
- a. A book with a wholesale price of \$3.75.
 - b. A movie with a wholesale price of \$6.00.
 - c. A music CD with a wholesale price of \$5.75.
 - d. A calendar with a wholesale price of \$4.80

Item	Percent markup
Books	60%
Music	40%
Movies	75%
Calendars	55%

8. A florist buys flowers from a wholesaler. He pays \$1.30 per rose and \$1.45 per lily. If the florist has a markup of 20% on all his flowers, find the retail price of:
- a. a rose.
 - b. a lily.

9. A general store buys carrots at a wholesale price of c dollars, and adds a markup of p percent. Write an expression for the retail price of the carrots.
-

Lesson
8.2.2

Tips, Tax, and Commission

California Standards: Number Sense 1.3, 1.7

Example

Matt goes out for dinner. His meal costs \$17.50 and he wants to leave a 10% tip.

- a. How much tip should he leave?
- b. How much money should he leave altogether?

Solution

 10 percent is the same as 10 hundredths.

- a. $\frac{10}{100} \times 17.50 = 1.75$, so his tip should be **\$1.75**.
- b. $17.50 + 1.75 = 19.25$, so he should leave **\$19.25** altogether.

1. A family of five has a restaurant bill totaling \$42.50. How much tip will they leave if they leave:

- a. a 10% tip?
- b. a 20% tip?
- c. a 16% tip?

2. a. The Kilcullin's lunch costs \$27 and they leave a 15% tip.

How much money do they leave altogether?

- b. The Heinbecker's dinner costs \$85 and they leave a 20% tip.

How much money do they leave altogether?

3. Simona spends \$15.89 on lunch. She wants to leave a 15% tip.

Estimate the amount of tip she should leave using mental math.

4. If the sales tax rate is 7% then determine the amount of tax on:

- a. a grocery bill of \$134.
- b. a hot tub that costs \$599.

5. Find the price of the following products after tax if the tax rate is 8%.
- a. A bag of groceries that cost \$13 before tax.
 - b. A catcher's mitt priced at \$55 before tax.
 - c. A jacket priced at \$89.50 before tax.
6. Ria's dinner costs \$48 before tax.
- a. She wants to leave a 20% tip before tax. Find the amount of tip she should leave.
.....
 - b. The tax rate is 6.5%. Find the amount of tax that needs to be added to the dinner bill.
.....
7. Find the tax rate charged on the following purchases.
- a. Nadia buys a \$24 alarm clock and pays an extra \$1.44 in tax.
 - b. Carlos buys a \$350 chair and pays an extra \$24.50 in tax.
 - c. Kyle buys a \$96 jacket and pays an extra \$8.16 in tax.
8. Britney is a realtor. She gets 4.5% commission on the sale price of any house.
How much commission would she be paid for selling a house for:
- a. \$180,000?
 - b. \$230,000?
 - c. \$980,600?
9. Jorge is a car salesman. He earns a commission of 2.6% of the pre-tax price of any car that he sells.
Last week he sold three cars in total, each for \$30,000 before tax.
- a. Calculate the amount of commission that Jorge earned last week.
 - b. If the sales tax rate is 6.5%, how much did each of the cars cost after tax?
 - c. How much commission would Jorge have earned on a car with a pre-tax price of \$30,000 if he received 2.6% of the price of the car after tax?
.....

Lesson
8.2.3

Profit

California Standards: Number Sense 1.3, 1.7

1. Jenny made buttons for a fund-raiser. Each button cost \$1.50 to make and was sold for \$4.35. How much profit did Jenny make on each button?
2. An online toy store had a profit of \$82,000 and a revenue of \$174,000 in a year. What were the company's expenses for that year?
3. The table shows the revenue and expenses of a golf course during the month of May.

	Cost	Revenue or Expense?
Lawn care	\$560	
Members fees	\$3475	
Advertising	\$1500	
Employment costs	\$2300	
Restaurant income	\$2750	

- a. Complete the table to show whether each item is a revenue or an expense.
- b. Find the profit for the golf course for the month of May.

Example

A general store makes a profit of \$8000 on total sales of \$25,000. What percent profit does the store make?

Solution

$$\frac{8000}{25,000} \times 100 = 32\%$$

4. Find the percent profit in each of the following cases:
 - a. A circus has ticket sales of \$600,000, and makes a profit of \$126,000.
 - b. A sports company makes a profit of \$180,000 on total sales of \$500,000.
 - c. A lemonade stand makes a profit of \$39 by selling \$65 worth of lemonade.
 - d. A small record company makes a profit of \$400,000 on total sales of \$1,600,000.

5. A music store charges \$13 for a certain CD, on which they earn a profit of \$3.25. They charge \$20 for another CD, on which they earn a profit of \$4.25.

a. What percent profit does the store make on a \$13 CD?

.....

b. What percent profit does the store make on a \$20 CD?

.....

c. Which of the two CDs makes the store a higher percent profit?

.....

6. A florist made a profit of \$2990 on sales of \$4600 during May, a profit of \$3200 on sales of \$5600 during June, and a profit of \$6018 on sales of \$10,300 in July. In which month did the florist make the highest percent profit? Show your work.

.....

.....

.....

7. A computer company's profit has decreased by 18% since the previous year. If the previous year's profit was \$278,000, find the profit made this year.

.....

8. The table shows the profits made by a sales team over 4 months.

a. Find the percent decrease in profit between months 1 and 2.

.....

b. Find the percent increase in profit between months 3 and 4.

.....

c. If the total sales for the four months were \$60,000, find the percent profit over this time.

.....

Month	Profit
1	\$4000
2	\$2300
3	\$6000
4	\$7200

9. Roberto's school puts on a play. Their expenses are: \$100 on costumes and set design, \$50 on advertising, and \$60 on refreshments. If they plan to sell 200 tickets and want to make a 20% profit, how much should they charge for each ticket?

.....

Lesson
8.2.4

Simple Interest

California Standards: Number Sense 1.3, 1.7

1. Simple interest can be calculated using the formula $I = Prt$.
What do the I and P in this formula represent?


.....

.....

Example

Jeremy deposits \$400 in a savings account that pays a simple interest rate of 5.5% per year.
How much interest will he get in 3 years?

Solution

$P = 400$, $r = 0.055$, $t = 3$,  Use the simple interest rate formula $I = Prt$.
so $I = Prt = 400 \times 0.055 \times 3 = \66

2. Calculate the amount of interest that would be earned for each of the following.
- a. \$5000 in an account that pays a simple interest rate of 4% per year for 3 years.
 - b. \$7800 in an account that pays a simple interest rate of 5% per year for 4 years.
 - c. \$12,000 in an account that pays a simple interest rate of 8% per year for 2 years.
3. Calculate the total amount in the following accounts after the time given.
- a. \$750 in an account that pays a simple interest rate of 5.2% per year for 4.5 years.
 - b. \$6000 in an account that pays a simple interest rate of 2.8% per year for 24 months.
 - c. \$38,000 in an account that pays a simple interest rate of 6.1% per year for 18 months.
4. Pauline took out a loan for \$6000 over 2.5 years.
If the simple interest rate is 7% per year, how much interest will Pauline have to pay?

.....

5. Elyssa borrows \$2500 from a bank at a simple interest rate of 12.3% per year. How much interest will she pay in 9.5 years?

.....

6. Tristan deposits \$2500 in an account that pays 7.4% simple interest per year. Azael deposits \$1950 in an account that pays 9.5% simple interest per year. Who will earn more interest over 2 years? Show your work.

.....

.....

7. Malcolm has a savings account that pays a simple interest rate of 10% per year. If he earns \$896 in a year, how much money did Malcolm invest?

.....

8. Find the amount that each person borrowed, given the information below.

a. Alana borrows money at a simple rate of 4% per year. After 2 years she has paid \$112 interest.

.....

b. Paco borrows money at a simple rate of 5% per year. After 1 year he has paid \$190 interest.

.....

c. Alsa pays a simple rate of 8.2% per year on her loan. After 3 years she has paid \$1476 interest.

.....

d. Brad pays a simple rate of 9.3% per year on a loan. After 2 years he has paid \$1421.04 interest.

.....

9. Gareth deposits \$1500 in an account with a simple interest rate of 6% per year for 2 years. He then moves all the money in the account (including the interest that he has earned) to an internet account with a simple interest rate of 9.4% per year. How much money will be in Gareth's internet account after a year? Show your work.

.....

.....

.....

.....

Lesson
8.2.5

Compound Interest

California Standards: Number Sense 1.3, 1.7

Example

Hermione invests \$25 in an account that gives 10% interest per year compounded quarterly. Use the compound interest formula to find her account balance after 1 year. Give your answer to the nearest cent.

Solution

The compound interest formula is $A = P(1 + rt)^n$.

Remember: A = amount in account, P = principal,
 r = rate of interest, t = time between each
interest payment in years,
 n = number of interest payments made.

$$P = 25, r = 0.1, t = 0.25, n = 4$$

$$\text{so } A = 25(1 + (0.1 \times 0.25))^4$$

$$= 25 \times (1.025)^4$$

$$= \$27.60 \text{ to the nearest cent.}$$

Interest is paid quarterly so in 1 year there will be 4 payments made.

- Mr. Heinzman deposits \$3800 in a savings account. If the interest is compounded quarterly at a rate of 5% per year then find his account balance after:
 - 3 months
 - 6 months
 - 1 year

.....
- Mrs. Bilderback deposits \$6000 in a savings account that gives an interest of 4% a year, compounded annually. How much will Mrs. Bilderback have in her account:
 - after 1 year?
 - after 3 years?
- The compound interest formula is $A = P(1 + rt)^n$. Use this formula with the following values to calculate A , the amount in an $r\%$ per year interest account, that is compounded every t years, after n interest payments have been made:
 - $P = \$750, r = 6.5\%, t = 0.25, n = 1$
 - $P = \$1800, r = 8\%, t = 0.5, n = 8$
 - $P = \$17,000, r = 9\%, t = 1, n = 12$
 - $P = \$120,000, r = 3.5\%, t = 1, n = 9$

4. Dolores deposits \$674 in a savings account with an interest rate of 4.5% per year, compounded quarterly. Find how much money will be in her account after:

a. 6 months b. 1 year
c. 3 years d. 10 years

5. Kenzie gets a loan of \$2450 to buy a car. The rate of the loan is 6.8% per year, which is compounded monthly. If she makes no repayments within the first year, find how much she owes after:

a. 3 months b. 6 months
c. 8 months d. 1 year

6. Paco deposits \$3,560,000 in an account that pays an interest rate of 7.35% per year, compounded quarterly. How much will be in his account after 18 years (to the nearest dollar)?

7. Explain the difference between compound interest and simple interest.

.....
.....
.....

8. \$3400 is deposited at a bank at a rate of 4% per year.

a. Complete the table to show the account balance over two years for a simple interest rate and an interest rate compounded annually.

b. Which account will have the most in it at the end of two years?

.....

Time in years	Simple interest	Compound interest
1		
2		

9. Jake is going traveling in 2 years' time and wants to put his savings of \$5860.76 in a high interest account until then. He is choosing between two accounts, one that offers a simple interest rate of 8.3%, and another account that offers an interest rate of 8.3%, compounded annually.

a. Without doing any calculations, which account should Jake put his money into?

.....

b. Find what the difference would be between the account balances by the time Jake goes traveling.




.....

**Lesson
8.3.1****Rounding****California Standards: Mathematical Reasoning 2.7****Example**

Round 64.38 to the nearest:

- a. ten
- b. tenth
- c. whole number

Solution

- a. 60  round down as the number to the right (in the ones place) is 4
- b. 64.4  round up as the number to the right (in the hundredths place) is 8
- c. 64  round down as the number to the right (in the tenths place) is 3

1. Round the following numbers to the nearest whole number.

- a. 7.2 b. 163.8 c. 99.5 d. 48.47
- e. 0.9 f. 71.39 g. 6.15 h. 423.9

2. Round the following numbers to the nearest tenth.

- a. 0.12 b. 9.876 c. 80.53 d. 7.046
- e. 2.872 f. 67.09 g. 376.16 h. 4.947

3. Round the following numbers to the size given.

- a. 5432.01 to the nearest hundred. b. 623.91 to the nearest ten.
- c. 0.7682 to the nearest tenth. d. 99.9 to the nearest whole number.
- e. 734,090 to the nearest 1000. f. 31.09357 to the nearest 1000th.

4. Michelle is rounding 5.31874302 to the nearest 1000th in her math homework. She said, “the digit to the left is 1, so the answer must be rounded down to 5.318.” Explain what Michelle has done wrong.

5. Rio has an allowance of \$20 per week. What is Rio's allowance per day?
Round your answer to the nearest cent.

.....

6. Round the following numbers to 1 decimal place.

a. 2.34 b. 16.01

c. 1.26 d. 51.37

e. 7.77 f. 8.219

g. 4.05 h. 1.009

7. Round the following numbers to the size given.

a. 9.816 to 2 decimal places.

b. 2.641 to 1 decimal place.

c. 50.6492 to 3 decimal places.

d. 18.731 to 2 decimal places.

e. 679.64522 to 4 decimal places.

f. 65,328,108.73 to 0 decimal places.

8. The value of π to 9 decimal places is 3.141592654.
Round the value of π to:

a. 4 decimal places.

b. the nearest tenth.

c. the nearest thousandth.

9. Solve the following calculations. Round your answers to the size given.

a. Divide 8.9 by 6.5. Give your answer to 3 decimal places.

b. Divide 15.7 by 2.3. Give your answer to 1 decimal place.

c. Multiply 654 by 856. Give your answer to the nearest thousand.

d. Divide 8900 by 7.8. Give your answer to 1 decimal place.

Lesson
8.3.2

Rounding Reasonably

California Standards: Mathematical Reasoning 2.7, 3.1

Example

Mei Mei wants to give a cereal bar to each of her 34 classmates.
If the cereal bars only come in packs of 3, how many packs must she buy?

Solution

$$34 \div 3 = 11.333 \text{ (to 3 decimal places)}$$

So she needs to buy 12 packs of cereal bars.



She can only buy packs of three cereal bars, so for everyone to have one she must round up.

1. Leo needs 12 apples to make apple pies. The apples at his local store only come in bags of 8.
How many bags of apples will he need to buy?

.....

2. Deanna is varnishing a 33 m^2 floor. A tin of varnish will coat 2 m^2 .
How many tins of varnish will she need to varnish the whole floor?

.....

3. Juan is making smoothies. He needs 2 bananas for each banana smoothie, 6 strawberries for each strawberry smoothie, and 5 chunks of pineapple for a pineapple smoothie.
He has 7 bananas, 28 strawberries, and 12 pineapple chunks.

a. How many banana smoothies can he make?

b. How many strawberry smoothies can he make?

c. How many pineapple smoothies can he make?

4. Michael wants to buy some boxes to store his 53 action figures.
He can fit 2 figures in a small box, and 5 figures in a large box.

a. How many small boxes would he need to buy to store his collection?

b. How many large boxes would he need to buy to store his collection?

5. Marcos is buying six-packs of juice boxes for a local basketball game.
- a. If he wants to buy 1 juice box for each of the 62 people attending the game, how many six-packs of boxes will he need to buy?
-

b. Daniel only has \$5 bills. If each six-pack of boxes costs \$5.25, how many \$5 bills will he need to buy the right amount of juice?

6. Jason has just got his allowance of \$25. He can't decide whether to spend it on CDs that cost \$12 each, or comic books that cost \$1.50 each.

a. How many CDs can Jason afford?

b. How many comic books can Jason afford?

7. A math teacher is ordering supplies for the classroom using the order form shown. She has a budget of \$900.

a. How many calculators can she afford?

b. How many protractors can she afford?

Item	Price	Number ordered
Protractor	\$1.30	
Calculator	\$32	

c. She decides she'd like to order 27 calculators and 30 protractors. Can she afford it?

8. Ed's dinner cost \$23.56, and he wants to leave at least a 15% tip. What is the smallest tip he should leave?
-

9. Rosita is saving up to buy her mom a birthday present costing \$56. If she saves \$8.50 a week, how many weeks will she need to save for?
-

Lesson

8.3.3 Exact and Approximate Answers

California Standards: Mathematical Reasoning 2.7, 2.8, 3.1

Example

The radius of a clock face is 4 inches.

- a. What is the exact area of the clock face?
 b. Approximate the area of the clock face to the nearest tenth of a square inch.

Solution

a. Area of a circle = $\pi r^2 = (4^2)\pi = 16\pi$

b. Area of a circle = $\pi r^2 = 16\pi = 50.3 \text{ in}^2$ to the nearest tenth of a square inch.

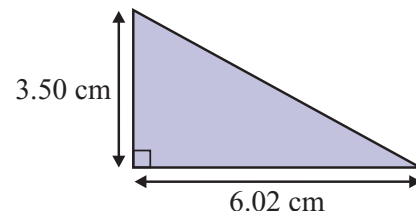
Leave π in the expression to give an exact answer.

1. Dione has drawn a square with an area of 18 in^2 .

- a. What is the exact side length of the square?
 b. Give the side length to the nearest inch.

2. a. Find the exact length of the hypotenuse of the triangle shown.

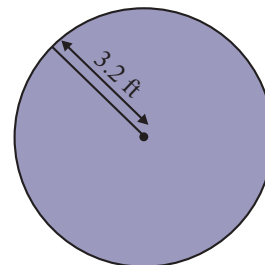
- b. Approximate the length of the hypotenuse to 1 decimal place.



3. The radius of a trampoline is 3.2 ft.

- a. What is the exact area of the trampoline?

- b. Give the area of the trampoline to the nearest ft^2 .



4. Liz went out cross-country running. She ran for 9.85 km and jogged for about 3 km. Liz said, "My total distance was 12.85 km." Is this an appropriate level of accuracy? Explain your answer.

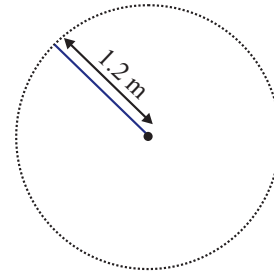
5. Round your answers to the following questions sensibly.

a. A hexagon has a perimeter of 896 ft. If all the sides are equal, find the side length.

b. Geraldine is making granola bars. If it costs her \$4.20 to make 11 bars, what is the lowest price she can charge for each bar to make at least as much as she spent?
.....

c. Pepe draws a rectangle with an area of 22.3 in^2 and a length of 15.6 in.
What is the width of the rectangle?
.....

6. Manuel is going into a store and so ties his dog to a lamppost outside. The dog's leash is measured as 1.2 m. If the leash is taut and the dog walks one complete circle, how far has the dog walked?
.....



7. State the smallest and largest values that each of the following measurements could have had before rounding. The first one has been done for you.

a. $x = 150$ inches rounded to the nearest ten inches.

$145 \text{ in.} \leq x < 155 \text{ in.}$

b. $y = 45$ miles to the nearest mile.

c. $x = 12.6$ in. rounded to 1 decimal place.

d. $z = 6$ ft rounded to the nearest ft.

e. $x = 90.72$ m rounded to 2 decimal places.

f. $x = 7.30$ m to the nearest cm.

8. Freya measures two strips of ribbon to be 12.2 cm long and 6.5 cm long, to the nearest 10 mm. Find the maximum and minimum sum of the two lengths of ribbon.
.....

9. Beila measures the side length of a cube to be 12.9 in.

a. What is the minimum area of a face of the cube?

b. What is the maximum volume of the cube?

Lesson

8.3.4

Reasonableness and Estimation**California Standards: Number Sense 1.3, Mathematical Reasoning 2.1, 2.3, 3.1****Example**

Charlie gets a 5% discount on the price of a \$5.20 toy plane.
He calculates the discounted price of the plane to be \$4.35.
Use estimation to check if he is right.

Solution

\$5.20 is around \$5.

10% of \$5 is \$0.50, so 5% of \$5 = \$0.25.

So the discounted price should be around $\$5 - \$0.25 = \$4.75$

His calculated price is too low, so he must have made an error in his calculation.

1. Decide whether the following answers are reasonable. In each case explain your answer.
 - a. Trudy has \$8 to split between her 6 children, so she decides to give them \$1.60 each.
.....
 - b. 15 people are going to a football game in four cars.
They want to split equally, and so decide to have 3.75 people in each car.
.....
2. Lucile is buying oranges from a grocery store. She has \$8 and the oranges cost 45 cents each.
How many oranges can Lucile afford?
.....
3. Paulo took an hour to walk 4 miles. He says it would take him around 1.2 hours to walk 12 ft.
Without any calculation explain whether Paulo is likely to be correct.
.....
4. Kyle's meal cost \$24.65. He wants to leave a 15% tip. He says he should leave about a \$3 tip.
Is he right? Estimate the tip he should leave.
.....

5. Grant wants to buy a pair of sneakers costing \$85.40. He will get a student discount of 12.5%. He calculates the discounted price of the sneakers, and says he will only have to pay \$74.725. Explain whether this is a sensible answer or not.

6. Harmony deposits \$58.30 in an account with a simple interest rate of 4.8% per year.

a. Estimate how much interest Harmony will have earned after 2 years.

b. Estimate how much interest Harmony will have earned after 50 years.

7. Keiran is saving up to buy a car that costs \$698.

If he saves \$52 a week, how many weeks will it take him to save enough to buy the car?

Check your answer using estimation. Show your work.

8. Evaluate each of the following expressions. Check your answers using estimation.

Answer

Estimate

a. 51×49

b. $3.3 \div 1.6$

c. $4.98 \div 5.14$

d. $9.156 \div 1.03$

e. $20.88 \div 7.17$

9. Cheryl cycled for half an hour at 15.62 miles per hour, and then 57 minutes at 10.29 miles per hour. Estimate how far she cycled.

Section 1.1

Lesson 1.1.1 — Variables and Expressions

1. a, b, d, and e should be circled.
2. 45,000 is not a variable expression because no variable is involved — 45,000 is just a constant.

Lesson 1.1.2 — Simplifying Expressions

1. a. $x + 2 + x = 2x + 2$
b. $x + x + x + 4 + 8 + x = 4x + 12$
c. $2x - 9 + x = 3x - 9$
2. a. $6x - 4y + 3y = 6x - y$
b. $4x + 15 + 2x - x + 9 = 5x + 24$
c. $j - k - j + 3k = 2k$

Lesson 1.1.3 — The Order of Operations

1. $144 + 3 \times 4 - 6 = 144 + 12 - 6 = 150$
2. a. $m \times (6 + 3) + 10 = m \times 9 + 10 = 9m + 10$
b. $4 + b \times 7 - 2 = 7b + 4 - 2 = 7b + 2$
3. Student B has the correct answer.

Lesson 1.1.4 — The Identity and Inverse Properties

1. a. 0
b. 1
2. a. -17
b. $\frac{1}{17}$
c. 6.2
d. $-\frac{1}{6.2}$

Lesson 1.1.5 — The Associative and Commutative Properties

1. Yes — $x = 4$ which gives $x - 4 = 4 - x = 0$.
2. a. $4p + (2p + 3) = (4p + 2p) + 3 = 6p + 3$
b. $67 + (5 + n) = (67 + 5) + n = 72 + n$
c. $-4(6m) = (-4 \times 6)m = -24m$
d. $8 \times (10 \times z) = (8 \times 10) \times z = 80z$
e. $12 + n + 2 + n = 12 + 2 + n + n = 14 + 2n$
f. $5 \times m \times 3 = 5 \times 3 \times m = 15m$
g. $-4 \times a \times 6 \times -b = -4 \times 6 \times a \times -b = 24ab$

Section 1.2

Lesson 1.2.1 — Writing Expressions

1. a. D
b. F
c. A
d. E
2. a. $92 + x$
b. $45 \div x$
c. $14 - x$
d. $1000x$

Lesson 1.2.2 — Equations

1. a. $6w$ is an **expression**
b. $12 = q - 8$ is an **equation**
c. $29 = \frac{r}{4}$ is an **equation**
d. $15 - t$ is an **expression**
e. $3(t - 45) = 125$ is an **equation**
2. a. $s + 23 = 9$ is a **variable equation**
b. $9 \div 3 = 3 \times 1$ is a **numeric equation**
c. $11 + d = e \div 9$ is a **variable equation**
d. $24 + 23 = 19 + 28$ is a **numeric equation**
e. $19 \times 3 = 342 \div 6$ is a **numeric equation**

Lesson 1.2.3 — Solving One-Step Equations

1. If you subtract a number from one side of an equation you must take the same number from the other side of the equation — so in this case, **11** needs to be subtracted from the right-hand side to balance the equation.
2. a. False. b. True. c. True. d. True.

Lesson 1.2.4 — Solving Two-Step Equations

1. $2g + 3 = 13$
 $2g = 13 - 3$
 $2g = 10$
 $g = 10 \div 2$
 $g = 5$
2. a. $5x + 1 = 26$
 $5x = 26 - 1$
 $5x = 25$
 $x = 25 \div 5$
 $x = 5$
b. $7y - 10 = 60$
 $7y = 60 + 10$
 $7y = 70$
 $y = 70 \div 7$
 $y = 10$

Lesson 1.2.5 — More Two-Step Equations

1. $\frac{2}{5}q = 32$
 $2q \div 5 = 32$
 $2q = 160$
 $q = 80$
2. a. $-7r - 5 = -12$
 $-7r = -7$
 $r = 1$
b. $\frac{1}{8}g + 1 = 99$
 $\frac{1}{8}g = 98$
 $g \div 8 = 98$
 $g = 784$

Lesson 1.2.6 — Applications of Equations

- Let m = the number of months the Piersons will have been paying fees for. Then they will have paid a total of $152m$ dollars in fees. There was also the enrollment fee of \$75 so after m months they will have paid a total of $75 + 152m$. So when the bill reaches \$683:

$$75 + 152m = 683$$

$$152m = 608$$

$$m = 4$$

So it will take **4 months** for the total bill to reach \$683.

- If the base length is 8 inches and the area is 36 square inches then:

$$36 = \frac{1}{2} \times 8h$$

$$36 = 4h$$

$$9 = h$$

$$h = 9$$

So the triangle is **9 inches** high.

Lesson 1.2.7 — Understanding Problems

- To solve this problem you would need to know the profits from the month before.
- To solve this problem you would need to know the cost of markers.

Section 1.3

Lesson 1.3.1 — Inequalities

- $5t = 25$ is an **equation**
 - $t + 25$ is an **expression**
 - $t < -5$ is an **inequality**
 - $w \geq 15$ is an **inequality**
- t is greater than 90
 - r is less than or equal to -32
 - 18 is greater than w
 - 0 is greater than or equal to z

Lesson 1.3.2 — Writing Inequalities

- $4 + x < 15$
 - $x \div 52 > 9$
 - $16 \leq 23 - 5$
 - $5x \geq 8$
- $x + 17 > 29$
 - $-9x \leq 28$
 - $-15 < \frac{x}{10}$
 - $x - 12 \geq 45$

Lesson 1.3.3 — Two-Step Inequalities

- $\frac{x}{2} + 10 \geq 58$
 - $4x + 7 < -23$
 - $17 \geq \frac{x}{3} + 43$

- The sum of 8 multiplied by a number, x , and 2 is greater than 50.
 - Fifteen minus half a number, x , is greater than or equal to 12.
 - Five less than twice a number, x , is less than 0.

Section 2.1

Lesson 2.1.1 — Rational Numbers

- $\frac{7}{1}$
 - $\frac{-12}{1}$
 - $\frac{14}{10}$
 - $\frac{58}{100}$
 - $\frac{175}{100}$
- $\sqrt{2}$ equals 1.4142135.....
 “The decimal does not repeat or terminate, so it’s an irrational number.”
 Or: “The decimal cannot be expressed as $\frac{p}{q}$ where p and q are integers”

Lesson 2.1.2 — Converting Terminating Decimals to Fractions

- Five and eight tenths or Five and four fifths
 - Six and two hundredths or Six and one fiftieth
 - Nine and forty five hundredths or Nine and nine twentieths

Fraction	Decimal	Terminating or Repeating
$\frac{52}{100}$	0.52	Terminating
$\frac{4}{11}$	0.$\overline{36}$	Repeating
$\frac{1}{2}$	0.5	Terminating
$\frac{904}{1000}$ or $\frac{113}{125}$	0.904	Terminating
$\frac{43}{10}$	4.3	Terminating

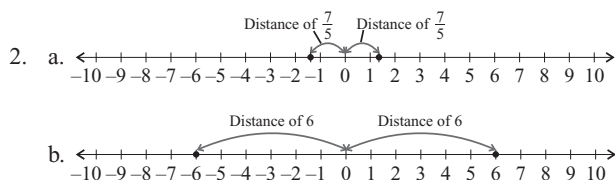
Lesson 2.1.3 — Converting Repeating Decimals to Fractions

- If the digits after the decimal point in two numbers are the same, then you can subtract one from the other and the decimal part of the number disappears.
- $$\begin{array}{r} 14.\overline{72} \\ - 2.\overline{72} \\ \hline 12.00 \\ 14.\overline{72} - 2.\overline{72} = 12 \end{array}$$

Section 2.2

Lesson 2.2.1 — Absolute Value

- $|2| = 2$
 - $|-5.5| = 5.5$
 - $|-130| = 130$
 - $\left|\frac{2}{5}\right| = \frac{2}{5}$



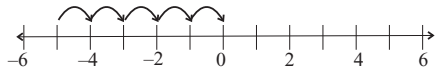
Lesson 2.2.2 — Using Absolute Value

- $|25 - 18| = |7| = 7$
 - $|-1.4 - 2.6| = |-4| = 4$
 - $|7 - (-3)| = |10| = 10$
- $|12 - (-4)| = |16| = 16$

Section 2.3

Lesson 2.3.1 — Adding and Subtracting Integers and Decimals

- $5 + -8 = -3$
 - $6 + (-2) = 4$
 - $-18 + (-1) = -19$
 - $-5 + (-5) = -10$
 - $-15 + 7 = -8$
 - $4 + (-10) = -6$
 - $-13 + 0 = -13$
 - $-7 + 4 + (-3) = -6$
 - $12 + 10 + (-17) = 5$
 - $3 + (-8) + 12 = 7$
- The sum is equivalent to starting at -5 and moving 5 units right, which takes you to 0.



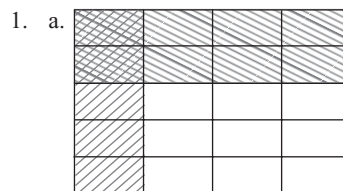
- The result of adding a number and its additive inverse is zero.

Lesson 2.3.2 — Multiplying and Dividing Integers

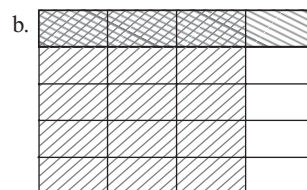
- $-7 \times 4 = -28$
 - $-1 \times (-11) = 11$
 - $9 \times 5 = 45$
 - $3 \times (-3) = -9$
 - $5 \times (-6) \times 2 = -30 \times 2 = -60$
 - $-3 \times (-10) \times (-2) = 30 \times (-2) = -60$
 - $-9 \times 0 \times 4 = 0 \times 4 = 0$
- $-14 \div 2 = -7$
 - $-90 \div -3 = 30$

- $64 \div 8 = 8$
- $-36 \div -4 = 9$
- $55 \div -5 = -11$
- $72 \div -9 \div -2 = -8 \div -2 = 4$
- $-45 \div -5 \div -3 = 9 \div -3 = -3$

Lesson 2.3.3 — Multiplying Fractions



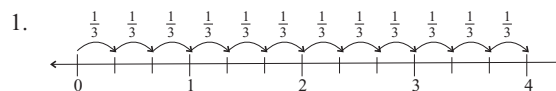
$$\frac{1}{4} \times \frac{2}{5} = \frac{2}{20} = \frac{1}{10}$$



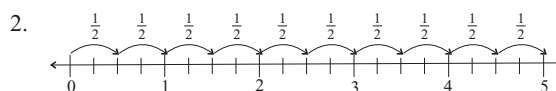
$$\frac{1}{5} \times \frac{3}{4} = \frac{3}{20}$$

- $-\frac{7}{10} \times \frac{2}{6} = -\frac{14}{60} = -\frac{7}{30}$
 - $-\frac{2}{5} \times -\frac{4}{13} = \frac{8}{65}$
 - $\frac{6}{7} \times \frac{1}{5} = \frac{6}{35}$
 - $-8 \times \frac{3}{16} = -\frac{24}{16} = -\frac{3}{2}$
 - $-\frac{7}{2} \times -\frac{4}{9} = \frac{28}{18} = \frac{14}{9}$

Lesson 2.3.4 — Dividing Fractions



$$4 \div \frac{1}{3} = 12$$



$$5 \div \frac{1}{2} = 10$$

Lesson 2.3.5 — Common Denominators

- 3×3
 - $2 \times 2 \times 3$
 - 17 doesn't factorize
 - 3×5
- $3 \times 3 \times 3 \times 3$
 - $3 \times 3 \times 5$
 - 3×13
 - $2 \times 2 \times 7$
 - $2 \times 2 \times 2 \times 2 \times 2 \times 2$
 - $3 \times 3 \times 3 \times 5$

Lesson 2.3.6 — Adding and Subtracting Fractions

- These two fractions have a common denominator of 15.
 $\frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$
 $\frac{12}{15}$ and $\frac{7}{15}$
 - These two fractions have a common denominator of 36.
 $\frac{5}{12} = \frac{5 \times 3}{12 \times 3} = \frac{15}{36}$ and $\frac{5}{18} = \frac{5 \times 2}{18 \times 2} = \frac{10}{36}$
 $\frac{15}{36}$ and $\frac{10}{36}$
 - These two fractions have a common denominator of 72.
 $\frac{11}{36} = \frac{11 \times 2}{36 \times 2} = \frac{22}{72}$ and $\frac{7}{24} = \frac{7 \times 3}{24 \times 3} = \frac{21}{72}$
 $\frac{22}{72}$ and $\frac{21}{72}$
- These two fractions have a common denominator of 15.
 $\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$ and $\frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$
 $\frac{10}{15} + \frac{12}{15} = \frac{10+12}{15} = \frac{22}{15}$

Lesson 2.3.7 — Adding and Subtracting Mixed Numbers

- $4\frac{3}{8} = \frac{(4 \times 8) + 3}{8} = \frac{35}{8}$
 - $7\frac{2}{7} = \frac{(7 \times 7) + 2}{7} = \frac{51}{7}$
 - $2\frac{11}{12} = \frac{(2 \times 12) + 11}{12} = \frac{35}{12}$
 - $7\frac{1}{5} = \frac{(7 \times 5) + 1}{5} = \frac{36}{5}$

$$2. \quad a. \quad 2\frac{2}{3} = \frac{(2 \times 3) + 2}{3} = \frac{8}{3} \text{ and } 5\frac{3}{4} = \frac{(5 \times 4) + 3}{4} = \frac{23}{4}$$

$$\frac{8}{3} + \frac{23}{4} = \left(\frac{8 \times 4}{3 \times 4}\right) + \left(\frac{23 \times 3}{4 \times 3}\right) = \frac{32}{12} + \frac{69}{12} = \frac{101}{12}$$

$$b. \quad 3\frac{1}{3} = \frac{(3 \times 3) + 1}{3} = \frac{10}{3} \text{ and } 6\frac{3}{8} = \frac{(6 \times 8) + 3}{8} = \frac{51}{8}$$

$$\frac{10}{3} + \frac{51}{8} = \left(\frac{10 \times 8}{3 \times 8}\right) + \left(\frac{51 \times 3}{8 \times 3}\right) = \frac{80}{24} + \frac{153}{24} = \frac{233}{24}$$

Section 2.4

Lesson 2.4.1 — Further Operations with Fractions

- $\frac{3}{4} + \frac{6}{2} \times \frac{8}{3}$
 $= \frac{3}{4} + \frac{6 \times 8}{2 \times 3}$
 $= \frac{3}{4} + \frac{48}{6}$
 $= \frac{3 \times 6}{4 \times 6} + \frac{48 \times 4}{6 \times 4}$
 $= \frac{18}{24} + \frac{192}{24}$
 $= \frac{210}{24} = \frac{35}{4}$
 - $\frac{20}{36} \div \frac{1}{3} - 2\frac{1}{6}$
 $= \frac{20}{36} \times \frac{3}{1} - \frac{(2 \times 6) + 1}{6}$
 $= \frac{20 \times 3}{36 \times 1} - \frac{13}{6}$
 $= \frac{60}{36} - \frac{13 \times 6}{6 \times 6}$
 $= \frac{60}{36} - \frac{78}{36}$
 $= -\frac{18}{36} = -\frac{1}{2}$

$$2. \quad a. \quad \left(3\frac{1}{3} + \frac{1}{4}\right) \times \frac{1}{2}$$

$$= \left(\frac{(3 \times 3) + 1}{3} + \frac{1}{4}\right) \times \frac{1}{2}$$

$$= \left(\frac{10 \times 4}{3 \times 4} + \frac{1 \times 3}{4 \times 3}\right) \times \frac{1}{2}$$

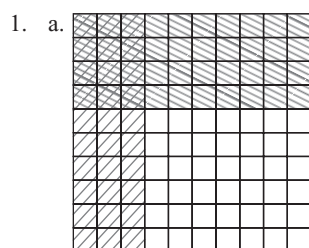
$$= \left(\frac{40}{12} + \frac{3}{12}\right) \times \frac{1}{2}$$

$$= \frac{43}{12} \times \frac{1}{2}$$

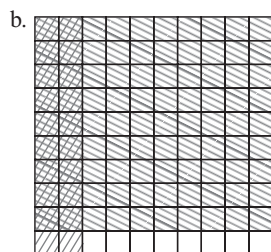
$$= \frac{43}{24}$$

$$\begin{aligned}
 \text{b. } & \left(2\frac{1}{5} + \frac{4}{5} \right) - 2 \times \frac{1}{6} \\
 & = \left(\frac{(2 \times 5) + 1}{5} + \frac{4}{5} \right) - \frac{2 \times 1}{6} \\
 & = \left(\frac{11}{5} + \frac{4}{5} \right) - \frac{2}{6} \\
 & = \frac{15}{5} - \frac{2}{6} \\
 & = 3 - \frac{2}{6} \\
 & = \frac{18}{6} - \frac{2}{6} \\
 & = \frac{16}{6} = \frac{8}{3}
 \end{aligned}$$

Lesson 2.4.2 — Multiplying and Dividing Decimals



$$0.3 \times 0.4 = 0.12$$



$$0.2 \times 0.9 = 0.18$$

2. a. $2 \times 1.3 = 2 \times \frac{13}{10}$ or $\frac{2}{1} \times \frac{13}{10}$

b. $3.4 \times -6.7 = \frac{34}{10} \times -\frac{67}{10}$

c. $1.02 \times 3.26 = \frac{102}{100} \times \frac{326}{100}$

d. $-4.6 \times 9.45 = -\frac{46}{10} \times \frac{945}{100}$

Lesson 2.4.3 — Operations with Fractions and Decimals

1. a. 0.35 is $\frac{35}{100} = \frac{7}{20}$. So, $\frac{7}{20} \times \frac{2}{5} = \frac{14}{100} = \frac{7}{50}$
 $\frac{2}{5}$ is $2 \div 5 = 0.4$. So, $0.35 \times 0.4 = 0.14$

b. 1.3 is $1\frac{3}{10} = \frac{13}{10}$. So, $\frac{1}{4} \times \frac{13}{10} = \frac{13}{40}$
 $\frac{1}{4}$ is $1 \div 4 = 0.25$. So, $0.25 \times 1.3 = 0.325$

2. a. Convert to fractions.

$$0.8 \text{ is } \frac{8}{10} = \frac{4}{5}$$

$$\frac{2}{3} - \frac{4}{5} = \frac{10}{15} - \frac{12}{15} = -\frac{2}{15}$$

b. Convert to fractions.

$$\text{First note that } 0.5 = \frac{1}{2}$$

Then calculate:

$$\frac{1}{2} \times \frac{1}{8} = \frac{1}{16}$$

Lesson 2.4.4 — Problems Involving Fractions and Decimals

1. Area = length \times width

$$\begin{aligned}
 &= 23\frac{1}{2} \times 10.3 \\
 &= \frac{47}{2} \times 10.3 \\
 &= (10.3 \div 2) \times 47 \\
 &= 5.15 \times 47 \\
 &= 242.05
 \end{aligned}$$

The area of the box is 242.05 in².

2. Area = length \times width

$$\begin{aligned}
 &= 6.5 \times 4\frac{1}{3} \\
 &= \frac{65}{10} \times \frac{13}{3} \\
 &= \frac{845}{30} \\
 &= \frac{169}{6}
 \end{aligned}$$

The area of the driveway is $28\frac{1}{6}$ yd².

Section 2.5

Lesson 2.5.1 — Powers of Integers

1.

	Base	Exponent
6^8	6	8
(-4)	-4	1
7^5	7	5
$(-6)^2$	-6	2

2. a. 3^2
b. 7^4
c. 8^1

Lesson 2.5.2 — Powers of Rational Numbers

1. a. $\left(\frac{9}{10}\right)^2 = \frac{9^2}{10^2} = \frac{81}{100}$
b. $\left(\frac{3}{8}\right)^3 = \frac{3^3}{8^3} = \frac{27}{512}$
c. $\left(\frac{1}{4}\right)^5 = \frac{1^5}{4^5} = \frac{1}{1024}$
d. $\left(-\frac{6}{5}\right)^4 = \frac{(-6)^4}{(5)^4} = \frac{1296}{625}$
2. a. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \left(\frac{1}{2}\right)^5$
b. $\frac{8}{9} \cdot \frac{8}{9} \cdot \frac{8}{9} = \left(\frac{8}{9}\right)^3$
c. $\frac{4}{3} \cdot \frac{4}{3} = \left(\frac{4}{3}\right)^2$

Lesson 2.5.3 — Uses of Powers

1. a. **True**, side length $= \sqrt{49} = 7$
b. **False**, $100^2 = 100 \times 100 = 10,000 \text{ cm}^2$
c. **False**, $\left(\frac{7}{8}\right)^2 = \frac{7^2}{8^2} = \frac{49}{64} \text{ ft}^2$
d. **True**, $\sqrt{0.01} = \sqrt{\frac{1}{100}} = \frac{\sqrt{1}}{\sqrt{100}} = \frac{1}{10} = 0.1 \text{ yd}$
2. a. Area $= \text{side length}^2 = 6^2 = 36 \text{ cm}^2$
b. Area $= \text{side length}^2 = 4.5^2 = \frac{45}{10} \times \frac{45}{10} = \frac{2025}{100} = 20.25 \text{ in}^2$
c. Area $= \text{side length}^2 = 6.2^2 = \frac{62}{10} \times \frac{62}{10} = \frac{3844}{100} = 38.44 \text{ ft}^2$

Lesson 2.5.4 — More on the Order of Operations

1. $15 + 2 \times 3^3$
 $= 15 + 2 \times 27$
 $= 15 + 54$
 $= 69$
2. $(6 - 9)^2 + 8^2 \div 2$
 $= (-3)^2 + 8^2 \div 2$
 $= 9 + 64 \div 2$
 $= 9 + 32$
 $= 41$

Section 2.6

Lesson 2.6.1 — Perfect Squares and Their Roots

1. a. **yes**, $15 \times 15 = 225$
b. **no**
c. **no**
d. **yes**, $7 \times 7 = 49$
e. **yes**, $100 \times 100 = 10,000$
2. **No**, a perfect square can never be a negative number (because two negative numbers multiplied together make a positive number.)

Lesson 2.6.2 — Irrational Numbers

1. a. $0.5 = \frac{5}{10} = \frac{1}{2}$
b. $19 = \frac{19}{1}$
c. $\sqrt{25} = 5 = \frac{5}{1}$
d. $0.\bar{6} = \frac{2}{3}$
2. a. $\sqrt{576} = 24 = \frac{24}{1}$
b. $\sqrt{0.81} = 0.9 = \frac{9}{10}$
c. $7.05 = \frac{705}{100} = \frac{141}{20}$
d. $-0.4 = -\frac{4}{10} = -\frac{2}{5}$

Lesson 2.6.3 — Estimating Irrational Roots

1. a. **irrational** (5 is not a perfect square number.)
b. **rational** (324 is a perfect square number: $18 \times 18 = 324$)
c. **irrational** (11 is not a perfect square number.)
2. a. **False**, $\sqrt{49} = 7$, so $\sqrt{48}$ must be less than 7.
b. **False**, $\sqrt{25} = 5$, so $\sqrt{34}$ must be greater than 5.
c. **True**, $\sqrt{64} = 8$, $\sqrt{81} = 9$, so $\sqrt{69}$ will be closer to 8 than 9.
d. **True**,
 $\sqrt{121} = 11$, $\sqrt{144} = 12$, so $\sqrt{130}$ will be between 11 and 12.

Section 3.1

Lesson 3.1.1 — Polygons and Perimeter

- This shape has a curved edge. The sides of a polygon are all straight.
 - This shape is not closed. A polygon is always a closed shape.
- Irregular
 - Regular
 - Irregular
 - Regular

Lesson 3.1.2 — Areas of Polygons

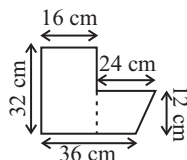
- Area = $bh = 12 \times 5 = 60 \text{ cm}^2$
 - Area = $lw = 2 \times 4.5 = 9 \text{ in}^2$
 - Area = $s^2 = 7^2 = 49 \text{ ft}^2$
 - Area = $\frac{1}{2}bh = \frac{1}{2} \times 21 \times 6 = 63 \text{ in}^2$
- Area = $s^2 = 20^2 = 400 \text{ in}^2$
 - Area = $lw = 5 \times 8 = 40 \text{ in}^2$
 - Area = $\frac{1}{2}bh = \frac{1}{2} \times 10 \times 16 = 80 \text{ cm}^2$
 - Area = $bh = 7 \times 13 = 91 \text{ yd}^2$

Lesson 3.1.3 — Circles

- $r = \frac{1}{2}d = \frac{1}{2} \times 120 \text{ mm} = 60 \text{ mm}$
 - $d = 2r = 2 \times 50 \text{ cm} = 100 \text{ cm}$
 - $d = 2r = 2 \times 14x = 28x$
 - $r = \frac{1}{2}d = \frac{1}{2}(8x + 6) = 4x + 3$
- $r = \frac{1}{2}d = \frac{1}{2} \times 36 = 18 \text{ mm}$
 - $d = 2r = 2 \times 9 = 18 \text{ cm}$
 - $d = 2r = 2 \times 4y = 8y$

Lesson 3.1.4 — Areas of Complex Shapes

- Split the shape into a rectangle and a trapezoid, e.g.

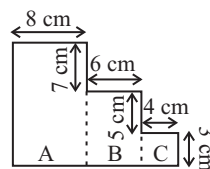


$$\text{Area of rectangle} = 16 \times 32 = 512 \text{ cm}^2$$

$$\text{Area of trapezoid} = \frac{1}{2} \times 12 \times (24 + (36 - 16)) = 264 \text{ cm}^2$$

$$\begin{aligned} \text{Total area} &= \text{area of rectangle} + \text{area of trapezoid} \\ \text{Total area} &= 512 + 264 = 776 \text{ cm}^2 \end{aligned}$$

- Split the shape into three rectangles, e.g.



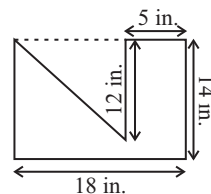
$$\text{Area A} = 8 \times (7 + 5 + 3) = 8 \times 15 = 120 \text{ cm}^2$$

$$\text{Area B} = 6 \times (5 + 3) = 6 \times 8 = 48 \text{ cm}^2$$

$$\text{Area C} = 4 \times 3 = 12 \text{ cm}^2$$

$$\text{Total area} = A + B + C = 120 + 48 + 12 = 180 \text{ cm}^2$$

- Treat the shape as a rectangle with a triangle taken out, i.e.

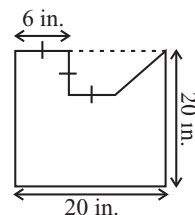


$$\text{Area of rectangle} = 14 \times 18 = 252 \text{ in}^2$$

$$\text{Area of triangle} = \frac{1}{2} \times (18 - 5) \times 12 = 13 \times 6 = 78 \text{ in}^2$$

$$\begin{aligned} \text{Total area} &= \text{area of rectangle} - \text{area of triangle} \\ &= 252 - 78 = 174 \text{ in}^2 \end{aligned}$$

- Treat the shape as a square with a trapezoid taken out, i.e.



$$\text{Area of square} = 20^2 = 400 \text{ in}^2$$

$$\begin{aligned} \text{Area of trapezoid} &= \frac{1}{2} \times 6 \times (6 + (20 - 6)) \\ &= 3 \times 20 = 60 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} \text{Total area} &= \text{area of square} - \text{area of trapezoid} \\ &= 400 - 60 = 340 \text{ in}^2 \end{aligned}$$

Lesson 3.1.5 — More Complex Shapes

- The shape is equal to three-quarters of a circle.

$$\text{Area} = \frac{3}{4}(\pi r^2) = \frac{3}{4}(3.14 \times 5^2) \approx 58.88 \text{ in}^2$$

- The shape is equal to a rectangle and a semicircle.

$$\text{Area of rectangle} = 3 \times 4 = 12 \text{ ft}^2$$

$$\text{Area of semicircle} = \frac{1}{2}(\pi r^2) = \frac{1}{2}(3.14 \times (3 \div 2)^2) \approx 3.53 \text{ ft}^2$$

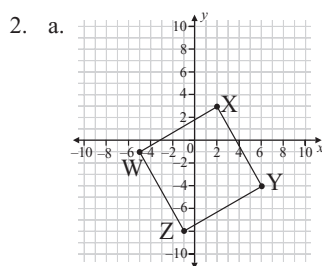
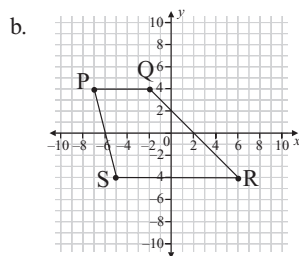
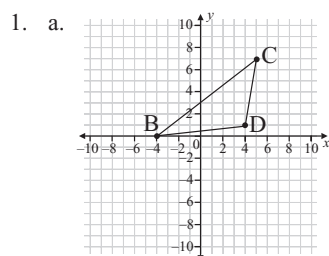
$$\text{Total area} \approx 12 + 3.53 = 15.53 \text{ ft}^2$$

Section 3.2

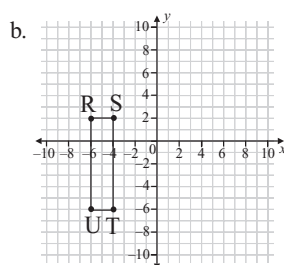
Lesson 3.2.1 — Plotting Points

- D
 - E
 - F
 - A
 - B
 - C
- False** An ordered pair is written (x, y) .
 - True** The origin, $(0, 0)$, is where the x - and y -axes cross.
 - False** Points in quadrant III are of the form $(-, -)$.
 - True** Points in quadrant I are of the form $(+, +)$.
 - False** The y -axis is vertical and the x -axis is horizontal.
 - True** All points with a y -coordinate of 0 lie on the x -axis.

Lesson 3.2.2 — Drawing Shapes in the Coordinate Plane



Square



Rectangle

Section 3.3

Lesson 3.3.1 — The Pythagorean Theorem

- The hypotenuse is the longest side.
 - 6 cm
 - p
 - 10 cm
 - 20 ft
- C cannot be used because it is not a right triangle.

Lesson 3.3.2 — Using the Pythagorean Theorem

- $x^2 = 10^2 + 6^2 \Rightarrow x = \sqrt{10^2 + 6^2}$
 - $10^2 = 6^2 + x^2 \Rightarrow x^2 = 10^2 - 6^2 \Rightarrow x = \sqrt{10^2 - 6^2}$
- The Pythagorean theorem states that $a^2 + b^2 = c^2$.
Substitute: $20^2 + 48^2 = c^2$
 $2704 = c^2$
 $c = 52$ **cm**
 - The Pythagorean theorem states that $a^2 + b^2 = c^2$.
Substitute: $24^2 + 45^2 = c^2$
 $576 + 2025 = c^2$
 $2601 = c^2$
 $c = 51$ **in.**
 - The Pythagorean theorem states that $a^2 + b^2 = c^2$.
Substitute: $a^2 + 36^2 = 45^2$
 $a^2 + 1296 = 2025$
 $a^2 = 729$
 $a = 27$ **cm**
 - The Pythagorean Theorem states that $a^2 + b^2 = c^2$.
Substitute: $18^2 + b^2 = 26^2$
 $324 + b^2 = 676$
 $b^2 = 352$
 $b = \sqrt{352} \approx 18.76$ **cm**

Lesson 3.3.3 — Applications of the Pythagorean Theorem

- $15^2 + b^2 = 39^2$
 $225 + b^2 = 1521$
 $b^2 = 1296$
 $b = 36$ **cm**
Area = $bh = 15 \times 36 = 540$ **cm²**
 - $(120 \div 2)^2 + b^2 = 68^2$
 $3600 + b^2 = 4624$
 $b^2 = 1024$
 $b = 32$ **in.**

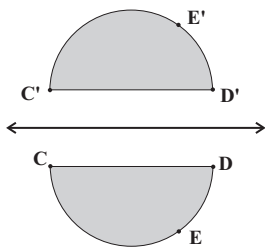
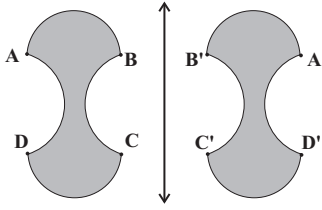
Area of triangle = $\frac{1}{2}bh = \frac{1}{2} \times 120 \times 32 = 1920$ **in²**
- $x^2 + 10^2 = 26^2$
 $x^2 + 100 = 676$
 $x^2 + 100 = 676$
 $x^2 = 576$
 $x = 24$ **ft**

Lesson 3.3.4 — Pythagorean Triples & the Converse of the Theorem

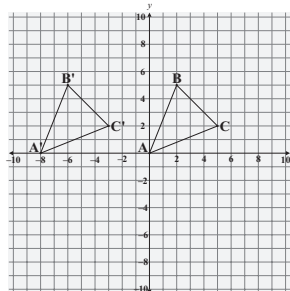
- Yes. The side lengths can be substituted into $a^2 + b^2 = c^2$.
 $18^2 + 80^2 = 82^2$
 $324 + 6400 = 6724$
 $6724 = 6724$
 - No — not all the sides are whole numbers.
 - No. The side lengths cannot be substituted into $a^2 + b^2 = c^2$ without creating a contradiction.
 $10^2 + 24^2 = 28^2$
 $100 + 576 = 784$
 676 does not equal 784
- Any whole number greater than one when multiplied by 6, 8, and 10 will produce a new Pythagorean triple.

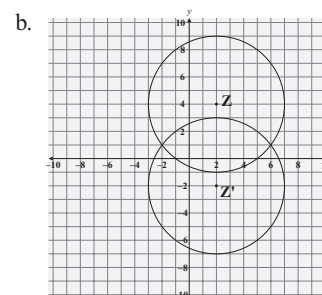
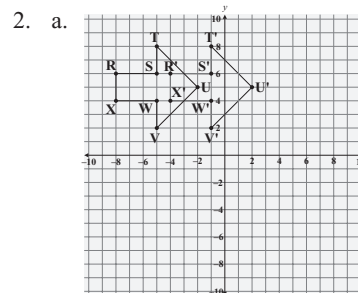
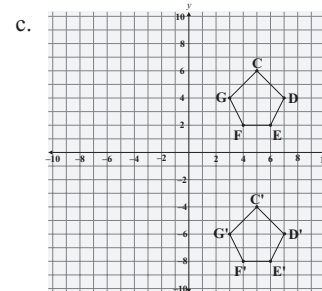
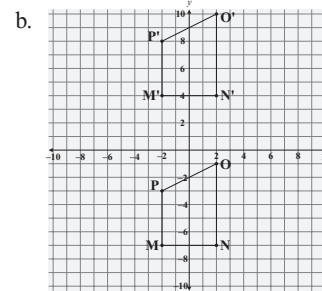
Section 3.4

Lesson 3.4.1 — Reflections

- 
 - 
- True
 - False
 - True

Lesson 3.4.2 — Translations

- 



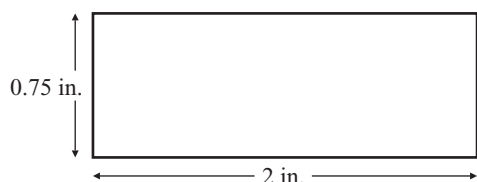
Lesson 3.4.3 — Scale Factor

- Compare side lengths: $\frac{2}{6} = \frac{1}{3}$
 - Compare diagonals PR and P'R' lengths: $\frac{6}{3} = 2$.
 - Compare side lengths: $\frac{8}{4} = 2$
 - Compare side lengths: $\frac{5}{10} = \frac{1}{2}$

2. Applying a scale factor changes the size of the figure whereas a reflection, translation, or rotation maintains the size of the figure.

Lesson 3.4.4 — Scale Drawings

1. a. $\frac{1}{4} = \frac{w}{3}$
 $4w = 3$
 $w = 0.75$ in.
 $\frac{1}{4} = \frac{h}{8}$
 $4h = 8$
 $h = 2$ in.
 The cabinets will measure **0.75 in. by 2 in.** in the scale drawing.
- b. The diagram must be a rectangle measuring 0.75 in. by 2 in.



2. a. The distance between Steelville and Monroe is 2 in. on the map.
 $\frac{1}{25} = \frac{2}{x}$
 $x = 50$
Steelville to Monroe is 50 miles.
- b. On the map, Monroe to Labtick is 1 in. and Labtick to Century is 3 in., so the total distance is 4 in.
 $\frac{1}{25} = \frac{4}{x}$
 $x = 100$
Monroe to Labtick to Century is 100 miles.
- c. On the map, Springfield to Steelville is 1.5 in. Steelville to Century is 1 in. So Springfield to Steelville is 0.5 in. farther than Steelville to Century.
 $\frac{1}{25} = \frac{0.5}{x}$
 $x = 12.5$
Springfield to Steelville is 12.5 miles farther than Steelville to Century.
- d. The cities would be closer on the map page.
 At 1 inch : 40 miles, 1 inch on the page represents a greater distance than at 1 inch : 25 miles.

Lesson 3.4.5 — Perimeter, Area, and Scale

1. a. Perimeter before scale = $3 + 4 + 5 = 12$ units
 Perimeter of image = $12 \times 2 = 24$ units
- b. Perimeter before scale = $6 + 4 + 2 + 2 + 4 + 2 = 20$ units
 Perimeter of image = $20 \times 0.25 = 5$ units

- c. Perimeter before scale = $4 + 6 + 4 + 6 = 20$ units
 Perimeter of image = $20 \times 3.5 = 70$ units

2. a. Area before scale = $\frac{1}{2} \times 6 \times 4 = 12$ units²

Area of image = $12 \times 2 \times 2 = 48$ units²

- b. Area before scale = $6 \times 3 = 18$ units²
 Area of image = $18 \times 5 \times 5 = 450$ units²

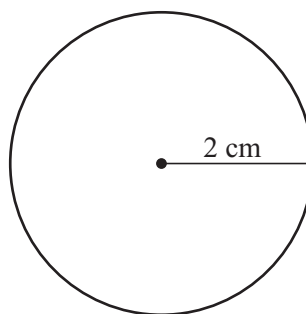
- c. Area before scale = $\frac{1}{2} \times 4 \times 2 + 4 \times 4 = 4 + 16 = 20$ units²

Area of image = $20 \times 25 \times 25 = 12,500$ units²

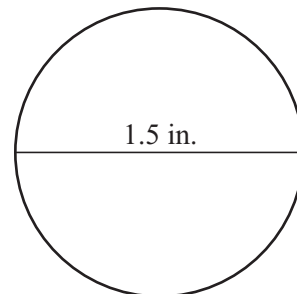
Section 3.5

Lesson 3.5.1 — Constructing Circles

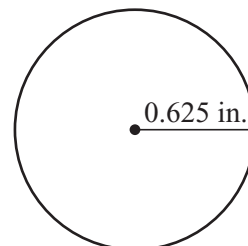
1. a.



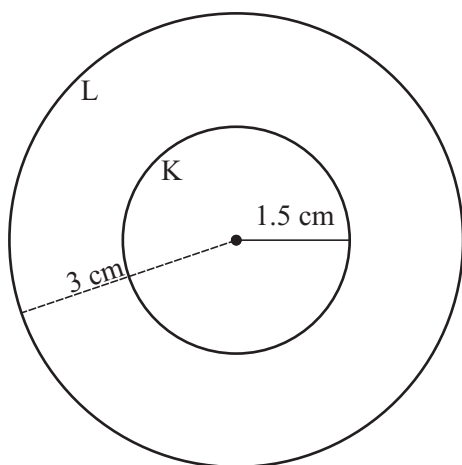
- b.



- c.



2. a.



- b. Circumference of K = $1.5 \times 2 \times \pi = 3\pi$ cm
 Area of K = $1.5^2\pi = 2.25\pi$ cm²
 Circumference of L = $3 \times 2 \times \pi = 6\pi$ cm
 Area of L = $3^2\pi = 9\pi$ cm²

Lesson 3.5.2 — Constructing Perpendicular Bisectors



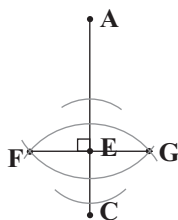
2. a. False. A line does not have a midpoint.
 A line segment has a midpoint.
 b. True
 c. True
 d. False. A line segment contains an infinite number of points.

Lesson 3.5.3 — Perpendiculars, Altitudes, and Angle Bisectors

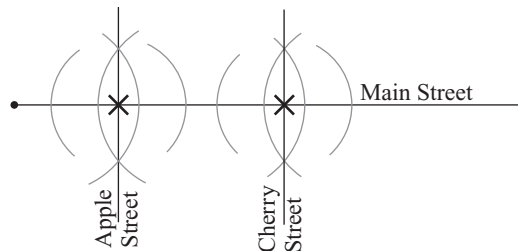
1. a.



b.



2.



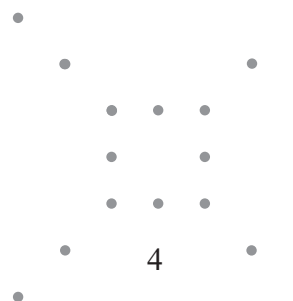
Section 3.6

Lesson 3.6.1 — Geometrical Patterns and Conjectures

1. For example:



Every even instance could be the previous instance with a dot added to each corner.



Every even instance could be the previous instance with one more dot added to each corner than in the last even instance.

2. For example, the sixth term is 32.

Lesson 3.6.2 — Expressions and Generalizations

1. a. For example, instance 4 will have 15 dots, instance 5 will have 21 dots.
 b. For example, the shape of the dots always forms a triangle, if n = the instance number, then each successive instance has $n + 1$ more dots, each term has symmetry about its diagonal.
2. a. For example, term 5 is 17, term 6 is 20.

- b. For example, each term is three more than the previous term, the terms are all integers, this pattern continues indefinitely.
- c. $3n + 2$ where $n = 1, 2, 3, \dots$

Section 4.1

Lesson 4.1.1 — Graphing Equations

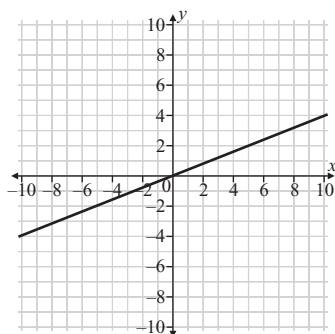
- Variables: y and x ,
Constants: m and b ,
Graph Type: **Straight Line**
- Yes**, at $x = 4, y = 5$
 - No**, at $x = 1, y = -1$
 - No**, at $x = 0, y = -3$
 - Yes**, at $x = -2, y = -7$
 - Yes**, the graph shows the equation $y = 2x - 3$.
Put a variety of x -values into the equation and check whether the corresponding y -value sits on the line.
For example, $(-2, -7)$, $(0, -3)$, $(2, 1)$, and $(6, 9)$ are all solutions to the equation and sit on the line.

Lesson 4.1.2 — Systems of Linear Equations

- false
 - false
 - true
 - false
- $2x + 3y = 45$ and $x - y = 3$

Lesson 4.1.3 — Slope

- undefined
 - negative
 - zero
 - positive
 - negative
- Any horizontal line
 - Any vertical line
 - Any line parallel to the graph shown:



Section 4.2

Lesson 4.2.1 — Ratios and Rates

- $5 : 2$ and $\frac{5}{2}$;
could also have answers like
“the ratio of pens to pencils is 2 to 5.”
- $7 : 4$

Lesson 4.2.2 — Graphing Ratios and Rates

- She worked 8 hours, so find 8 hours on the horizontal axis, go up to the graphed line, then across to the vertical axis find the amount earned in 8 hours: **\$320**
- Find the slope: the change in y divided by the change in x (this is also the unit rate). On this graph, this is
$$\frac{\text{distance traveled (miles)}}{\text{time taken (hours)}}$$
so the units will be miles per hour.
Choose two points on the line and find the vertical change (y) and the horizontal change (x). For example $(5, 250)$ and $(2, 100)$
Change in $y = 250 - 100 = 150$ miles
Change in $x = 5 - 2 = 3$ hours
So, the slope or unit rate = $\frac{150}{3} = \mathbf{50 \text{ miles/hour}}$

Lesson 4.2.3 — Distance, Speed, and Time

- $s = \frac{d}{t}$
 - $t = \frac{d}{s}$
- $s = 250/5 = 50$ miles per hour
 - $t = 450/60 = 7.5$ hours

Lesson 4.2.4 — Direct Variation

- $y = kx$ $3 = 12k$ so $k = \frac{3}{12} = \frac{1}{4}$ $y = \frac{1}{4}x$
 - $y = kx$ $15 = 5k$ so $k = \frac{15}{5} = 3$ $y = 3x$
 - $y = kx$ $136 = 68k$ so $k = \frac{136}{68} = 2$ $y = 2x$
- $y = kx + 8$ is not an equation of direct variation. x is not a multiple of y because the equation includes a constant term.
The graph of any equation with direct variation has a y -intercept of zero, but the equation $y = kx + 8$ has a graph with a y -intercept of 8.

Section 4.3

Lesson 4.3.1 — Converting Measures

- 1 : 2
10 : 1
1 : 16
1 : 1000
- The ratio of meters to kilometers is 1000 : 1.
Let x = the number of meters in 3 kilometers.
$$\frac{1000}{1} = \frac{x}{3}$$
$$1 \times x = 1000 \times 3$$
$$x = \mathbf{3000 \text{ meters}}$$
 - The ratio of liters to milliliters is 1 : 1000.
Let x = the number of liters in 500 milliliters.
$$\frac{1}{1000} = \frac{x}{500}$$
$$1000 \times x = 1 \times 500$$
$$x = 500 \div 1000 = \mathbf{0.5 \text{ liters}}$$
 - The ratio of grams to kilograms is 1000 : 1.
Let x = the number of grams in 0.7 kilograms.
$$\frac{1000}{1} = \frac{x}{0.7}$$
$$1 \times x = 1000 \times 0.7$$
$$x = \mathbf{700 \text{ grams}}$$

Lesson 4.3.2 — Converting Between Unit Systems

- 2.2 : 1 or 1 : 0.45
1 : 0.91
1 : 1.057
- The ratio of kilograms to pounds is approximately 1 : 2.2. Let x = the number of pounds in 4 kilograms.
$$\frac{1}{2.2} = \frac{4}{x}$$
$$1 \times x = 2.2 \times 4$$
$$x = \mathbf{8.8 \text{ pounds}}$$
 - The ratio of yards to meters is approximately 1 : 0.91.
Let x = the number of meters in 5 yards.
$$\frac{1}{0.91} = \frac{5}{x}$$
$$1 \times x = 0.91 \times 5$$
$$x = \mathbf{4.55 \text{ meters}}$$

Lesson 4.3.3 — Dimensional Analysis

- 600 words \div 3 minutes = 200 **words/minute**
 - 10 feet \times 15 feet = 150 **feet²**
 - 10 $\frac{\text{dollars}}{\text{hours}} \times 8 \text{ hours} = 80 \text{ dollars}$
 - 60 miles \div 40 $\frac{\text{miles}}{\text{hours}} = 60 \text{ miles} \times \frac{1 \text{ hour}}{40 \text{ miles}} = 1.5 \text{ hours}$

$$e. 2 \frac{\text{meters}}{\text{hours}} \times 24 \frac{\text{hours}}{\text{day}} = 48 \text{ meters/day}$$

- Number of people = 3
Number of days taken to build the treehouse = 4
 $3 \text{ persons} \times 4 \text{ days} = \mathbf{12 \text{ person-days}}$

Lesson 4.3.4 — Converting Between Units of Speed

- $\frac{1 \text{ foot}}{12 \text{ inches}}$ or $\frac{12 \text{ inches}}{1 \text{ foot}}$
 - $\frac{1 \text{ year}}{12 \text{ months}}$ or $\frac{12 \text{ months}}{1 \text{ year}}$
 - $\frac{3600 \text{ seconds}}{1 \text{ hour}}$ or $\frac{1 \text{ hour}}{3600 \text{ seconds}}$
 - $\frac{1 \text{ inch}}{2.54 \text{ cm}}$ or $\frac{2.54 \text{ cm}}{1 \text{ inch}}$
- 1 hour = 60 minutes
$$\frac{60 \text{ minutes}}{1 \text{ hour}} = 1$$
$$1.4 \text{ hours} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = 1.4 \times 60 \text{ minutes} = \mathbf{84 \text{ minutes}}$$
 - 1 mile \approx 1.6 kilometers
$$\frac{1.6 \text{ km}}{1 \text{ mile}} \approx 1$$
$$4 \text{ miles} \times \frac{1.6 \text{ km}}{1 \text{ mile}} = 4 \times 1.6 \text{ km} = \mathbf{6.4 \text{ km}}$$

Section 4.4

Lesson 4.4.1 — Linear Inequalities

- n is greater than seven.
 - m is less than or equal to 50.
 - g is greater than or equal to negative five.
 - x is less than negative 22.
- $y \leq 12$
 - $h > -4$
 - $b < 0$
 - $q \geq -1$

Lesson 4.4.2 — More on Linear Inequalities

- No** — adding a negative number doesn't reverse the sign of an inequality.
 - Yes** — dividing by a negative number does reverse the sign of an inequality.
 - Yes** — multiplying by a negative number (including negative fractions and negative decimals) does reverse the sign of an inequality.

2. a. $4r < 20$
 $4r \div 4 < 20 \div 4$
 $r < 5$
- b. $g \div 3 \geq 25$
 $g \div 3 \times 3 \geq 25 \times 3$
 $g \geq 75$
- c. $12w \leq 72$
 $12w \div 12 \leq 72 \div 12$
 $w \leq 6$
- d. $y \div 4 > 8$
 $y \div 4 \times 4 > 8 \times 4$
 $y > 32$

Lesson 4.4.3 — Solving Two-Step Inequalities

1. a. $4x + 5 < 21$
 $4x + 5 - 5 < 21 - 5$
 $4x < 16$
 $4x \div 4 < 16 \div 4$
 $x < 4$
- b. $3b - 12 \geq -6$
 $3b - 12 + 12 \geq -6 + 12$
 $3b \geq 6$
 $3b \div 3 \geq 6 \div 3$
 $b \geq 2$
- c. $(g \div 3) - 13 \leq -1$
 $(g \div 3) - 13 + 13 \leq -1 + 13$
 $(g \div 3) \leq 12$
 $(g \div 3) \times 3 \leq 12 \times 3$
 $g \leq 36$
2. a. The inequality should say that the product of the number of cars washed, c , and 8, minus 11, needs to be greater than or equal to 125. So the correct inequality is $8c - 11 \geq 125$, or **inequality i**.
- b. $8c - 11 \geq 125$
 $8c - 11 + 11 \geq 125 + 11$
 $8c \geq 136$
 $8c \div 8 \geq 136 \div 8$
 $c \geq 17$

Section 5.1

Lesson 5.1.1 — Multiplying with Powers

1. a. False. $6^3 \cdot 6^2 = 6^{3+2} = 6^5$
b. True.
c. False. $9^3 \cdot 9^7 = 9^{3+7} = 9^{10}$
d. False. The two bases are not the same, so you can't use the multiplication of powers rule.
e. False. $7^2 \cdot 7^6 = 7^{2+6} = 7^8$
f. True
2. a. $5^2 \cdot 5^3 = 5^{2+3} = 5^5$
b. $6^6 \cdot 6^2 = 6^{6+2} = 6^8$
c. $9^3 \cdot 9^7 \cdot 9^2 = 9^{3+7+2} = 9^{12}$
d. $3^3 \cdot 3 = 3^3 \cdot 3^1 = 3^{3+1} = 3^4$
e. $7^3 \cdot 7^9 = 7^{3+9} = 7^{12}$
f. $8^4 \cdot 8^0 = 8^{4+0} = 8^4$
g. $h^{11} \cdot h^7 = h^{11+7} = h^{18}$

Lesson 5.1.2 — Dividing with Powers

1. a. True
b. False. $4^8 \div 4^2 = 4^{8-2} = 4^6$
c. False. $9^8 \div 9^5 = 9^{8-5} = 9^3$
d. False. Convert 4^3 into 2^6 . $2^9 \div 2^6 = 2^3$
e. True
f. True
2. a. $6^8 \div 6^2 = 6^{8-2} = 6^6$
b. $8^6 \div 8^2 = 8^{6-2} = 8^4$
c. $7^3 \div 7 = 7^{3-1} = 7^2$
d. $2^5 \div 2^2 = 2^{5-2} = 2^3$
e. $9^{10} \div 9^7 = 9^{10-7} = 9^3$
f. $4^3 4^8 \div 4^9 = 4^{3+8} \div 4^9 = 4^{11} \div 4^9 = 4^{11-9} = 4^2$
g. $x^8 \div x^3 = x^{8-3} = x^5$

Lesson 5.1.3 — Fractions with Powers

1. a. $\left(\frac{1}{3}\right)^4 \times \left(\frac{1}{3}\right)^6 = \left(\frac{1}{3}\right)^{4+6} = \left(\frac{1}{3}\right)^{10}$
b. 11 is an odd number, so $\left(-\frac{3}{5}\right)^{11} = -\left(\frac{3}{5}\right)^{11}$. That means that
 $\left(-\frac{3}{5}\right)^{11} \times \left(\frac{3}{5}\right)^{13} = -\left(\frac{3}{5}\right)^{11} \times \left(\frac{3}{5}\right)^{13} = -\left(\frac{3}{5}\right)^{11+13} = -\left(\frac{3}{5}\right)^{24}$
c. $\left(\frac{x}{y}\right)^7 \times \left(\frac{x}{y}\right)^{12} = \left(\frac{x}{y}\right)^{7+12} = \left(\frac{x}{y}\right)^{19}$
2. a. $\left(\frac{1}{5}\right)^5 \div \left(\frac{1}{5}\right)^2 = \left(\frac{1}{5}\right)^{5-2} = \left(\frac{1}{5}\right)^3$
b. $\left(\frac{7}{8}\right)^4 \div \left(\frac{7}{8}\right)^3 = \left(\frac{7}{8}\right)^{4-3} = \left(\frac{7}{8}\right)^1 = \frac{7}{8}$
c. $\left(\frac{x}{y}\right)^{10} \div \left(\frac{x}{y}\right)^6 = \left(\frac{x}{y}\right)^{10-6} = \left(\frac{x}{y}\right)^4$

Section 5.2

Lesson 5.2.1 — Negative and Zero Exponents

1. It goes down by two.
2. a. $8^0 = 1$
b. $\left(\frac{1}{5}\right)^0 = 1$
c. $(-4)^0 = 1$
d. $(x+z)^0 = 1$

Lesson 5.2.2 — Using Negative Exponents

1. a. division of powers
b. multiplication of powers
2. $6^{-7} = \frac{1}{6^7} = \frac{1}{6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6}$

Lesson 5.2.3 — Scientific Notation

- $4.7 \times 10^5 = 4.7 \times 100,000 = \mathbf{470,000}$
 - $8 \times 10^8 = 8 \times 100,000,000 = \mathbf{800,000,000}$
 - $2.003 \times 10^{10} = 2.003 \times 10,000,000,000 = \mathbf{20,030,000,000}$
- $67,000,000,000,000 = 6.7 \times 10,000,000,000,000 = \mathbf{6.7 \times 10^{13}}$
 - $5,020,000,000 = 5.02 \times 1,000,000,000 = \mathbf{5.02 \times 10^9}$
 - $400,000,000 = 4 \times 100,000,000 = \mathbf{4 \times 10^8}$
 - $49,000 = 4.9 \times 10,000 = \mathbf{4.9 \times 10^4}$

Lesson 5.2.4 — Comparing Numbers in Scientific Notation

- 6×10^{-9} is the least because -9 is the least exponent.
- 3×10^{-5} , 7×10^{-4} , 2×10^3 , 2×10^5 , 7×10^5

Section 5.3

Lesson 5.3.1 — Multiplying Monomials

- Is a monomial.
 - Not a monomial.
 - Is a monomial.
 - Not a monomial.
 - Not a monomial.
- 54
 - 12
 - 1
 - $5^3 = 125$

Lesson 5.3.2 — Dividing Monomials

- $9g^8 \div 3g^5 = (9 \div 3)g^{8-5} = \mathbf{3g^3}$
 - $3v^6 \div 9v^2 = \frac{3}{9}v^{6-2} = \frac{1}{3}v^4 = \frac{v^4}{3}$
 - $-6t^5w^4 \div 12t^5w^5 = \frac{-6}{12}t^{5-5}w^{4-5} = -\frac{1}{2}t^0w^{-1} = -\frac{1}{2}w^{-1}$ or $\frac{-w^{-1}}{2}$
 - $\frac{a^6}{5} \div 5^2a^3c^4 = \frac{a^6}{5} \div 25a^3c^4 = \left(\frac{1}{5} \div 25\right)a^{6-3}c^{0-4} = \frac{1}{125}a^3c^{-4} = \frac{a^3c^{-4}}{125}$
 - $15w^5r^1 \div -15w^5r^1 = (15 \div -15)w^{5-5}r^{1-1} = -1w^0r^0 = \mathbf{-1}$
- $18j^3g^2 \div 6j^3g^1 = (18 \div 6)j^{3-3}g^{2-1} = 3j^0g^1 = \mathbf{3g}$, a monomial
 - $5d^6r^4 \div 10d^2r = \frac{5}{10}d^{6-2}r^{4-1} = \frac{1}{2}d^4r^3$, a monomial
 - $7m^6n \div 7m^8n = (7 \div 7)m^{6-8}n^{1-1} = 1m^{-2}n^0 = \mathbf{m^{-2}}$, not a monomial
 - $14g^6h^4t^7 \div 42g^9 = \frac{14}{42}g^{6-9}h^4t^7 = \frac{1}{3}g^{-3}h^4t^7$, not a monomial

Lesson 5.3.3 — Powers of Monomials

- $(7^2)^5 = (7 \times 7) \times (7 \times 7) \times (7 \times 7) \times (7 \times 7) \times (7 \times 7)$
 $= 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$
 - $(7^2)^5 = 7^{2 \times 5} = \mathbf{7^{10}}$
- $(5^4)^9 = 5^{4 \times 9} = 5^{36}$, so $x = \mathbf{36}$.

Lesson 5.3.4 — Square Roots of Monomials

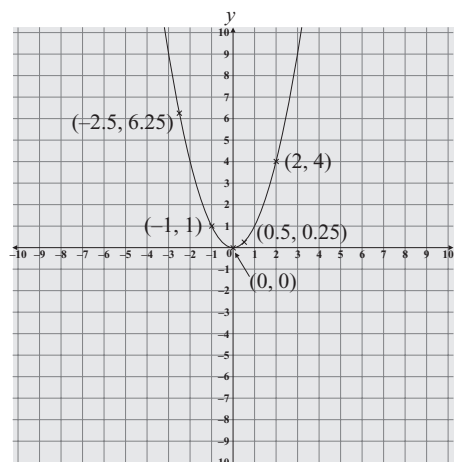
- 5
 - 7
 - v^2
 - $(n^4)^2$
- Austin and Brandon have been given money, so they must both have positive amounts. That means you can use the positive square roots.
 The square root of 144 is 12, so Austin has \$12.
 The square root of 16 is 4 and the square root of 81 is 9.
 So Brandon has a total of $4 + 9 = \$13$.
 That means that **Brandon** has **\$1 more** than Austin.

Section 5.4

Lesson 5.4.1 — Graphing $y = nx^2$

- The vertex is at $(0, 0)$.
 - When $x = -7$, $y = (-7)^2 = \mathbf{49}$.
 - When $x = 2.5$, $y = (2.5)^2 = \mathbf{6.25}$.
 - When $x = \frac{1}{4}$, $y = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$
 - $x = \pm\sqrt{81}$, so $x = \mathbf{9}$ or $\mathbf{-9}$.
 - $x = \pm\sqrt{900}$, so $x = \mathbf{30}$ or $\mathbf{-30}$.
 - $x = \pm\sqrt{0.36}$, so $x = \mathbf{0.6}$ or $\mathbf{-0.6}$.
 - $x = \pm\sqrt{0.25}$, so $x = \mathbf{0.5}$ or $\mathbf{-0.5}$.

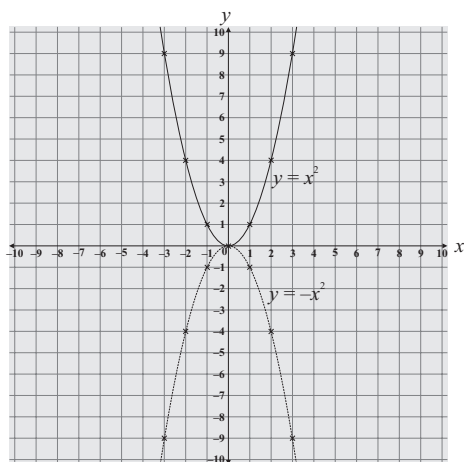
x	x^2
-2.5	6.25
-1	1
0	0
0.5	0.25
2	4



Lesson 5.4.2 — More Graphs of $y = nx^2$

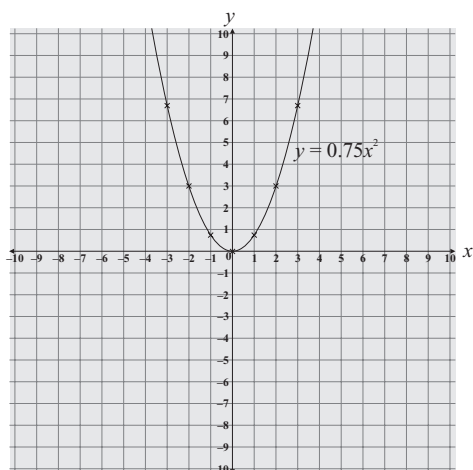
1.

x	x^2	$-x^2$
-2.5	6.25	-6.25
-1	1	-1
0	0	0
1	1	-1
2.5	6.25	-6.25



2.

x	$0.75x^2$
-3	6.75
-2	3
-1	0.75
0	0
1	0.75
2	3
3	6.75



Lesson 5.4.3 — Graphing $y = nx^3$

- False
 - False
 - False
 - True
- Decrease
 - Increase
 - Increase
 - Increase

Section 6.1

Lesson 6.1.1 — Median and Range

- Order the data set: {499, 506, 560, 687, 730, 780}
The set has six values, so the median is the average of the third and fourth values. $(560 + 687) \div 2 = \mathbf{623.5}$
 - Order the data set: {4.3, 4.9, 5.0, 5.0, 5.6, 6.8, 6.8}
The set has seven values, so the median is the fourth value, **5.0**.
 - Order the data set: {4, 7, 11, 12, 18, 19, 19}
The set has seven values, so the median is the fourth value, **12**.
 - Order the data set: {0.06, 0.1, 0.6, 0.7, 0.9, 1.6}
The set has six values, so the median is the average of the third and fourth values. $(0.6 + 0.7) \div 2 = \mathbf{0.65}$
- Order the known values of the data set: {12, 24, 30, 35, 40}
The complete set has six values, so the median, 28, is the average of the third and fourth values. The third and fourth values could be:
24 and 30: $(24 + 30) \div 2 = 27$
30 and 35: $(30 + 35) \div 2 = 32.5$
Neither of these give the right median, so x must be the third number.
(It can't be the fourth because the median is less than 30.)
30 and x : $(30 + x) \div 2 = 28$
 $30 + x = 28 \times 2$
 $30 + x = 56$
 $x = \mathbf{26}$

Lesson 6.1.2 — Box-and-Whisker Plots

- Maximum: 28, Minimum: 5
 - The data set has seven values, so the fourth value, **20**, is the median.
 - The upper half of the data set has three values, so the second value of the half, **25**, is the upper quartile of the data set.
 - The lower half of the data set also contains three values, so the second value, **8**, is the lower quartile of the data set.
- Maximum: 98, Minimum: 63
 - 85
 - It is impossible to tell the number of values in a data set from a box-and-whisker plot, but in this case you can tell that there are at least 5 values.

Lesson 6.1.3 — More on Box-and-Whisker Plots

- Class Y
 - Class X
- Team B (the median is lower)
 - Team A
 - Team A (median is close to 74)

Lesson 6.1.4 — Stem-and-Leaf Plots

- 6 9
7 2 2 5 7 8 9
8 0 4 7 7 8 8
9 0 1 3 5 5 7
- a. 28
b. 9

Lesson 6.1.5 — Preparing Data to Be Analyzed

- a. Order the data set: {185.6, 185.7, 186.3, 187.8, 188.0, 188.2, 188.7, 189.4, 190.1, 190.3, 190.6, 190.9, 191.7, 194.2}
The set has 14 values, so the median is the average of the seventh and eighth values. $(188.7 + 189.4) \div 2 = \mathbf{189.05}$
b. Range = $194.2 - 185.6 = \mathbf{8.6}$
c. The lower half contains seven values, so the fourth value, **187.8**, is the lower quartile.
d. The upper half contains seven values, so the fourth value, **190.6**, is the upper quartile.
- a. 2000: \$700,000
2005: \$800,000
b. 2000: \$450,000
2005: \$495,000
c. 2000: Order the data set: {450,000, 500,000, 500,000, 550,000, 600,000, 650,000, 700,000, 700,000}
The set has eight values, so the median is the average of the fourth and fifth values. $(550,000 + 600,000) \div 2 = \mathbf{\$575,000}$
2005: Order the data set: {495,000, 550,000, 600,000, 605,000, 660,000, 770,000, 800,000, 800,000}
The set has eight values, so the median is the average of the fourth and fifth values. $(605,000 + 660,000) \div 2 = \mathbf{\$632,500}$
d. 2000: The lower half of the set has four values, so the lower quartile is the average of the second and third values.
 $(500,000 + 500,000) \div 2 = \mathbf{\$500,000}$
2005: The lower half of the set has four values, so the lower quartile is the average of the second and third values.
 $(550,000 + 600,000) \div 2 = \mathbf{\$575,000}$
e. 2000: The upper half of the set has four values, so the upper quartile is the average of the second and third values.
 $(650,000 + 700,000) \div 2 = \mathbf{\$675,000}$
2005: The upper half of the set has four values, so the upper quartile is the average of the second and third values.
 $(770,000 + 800,000) \div 2 = \mathbf{\$785,000}$

Lesson 6.1.6 — Analyzing Data

- a. **True** (the 5th grade data has a lower minimum than the 6th grade data and a similar maximum)
b. **False** (the lower quartile is close to 60, so about 75% are taller than 60 inches.)
c. **True** (the lower quartile is approximately 58 and the median 60)
d. **True** (all the 6th grade measures except the maximum are higher than the respective 5th grade measures)
- Month A — January, Month B — June
The temperatures are generally higher in month B than in month A.

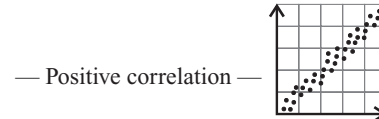
Section 6.2

Lesson 6.2.1 — Making Scatterplots

- The length of time a set of students studies and their math test scores.
- The heights of a set of students and the dates of their birthdays.

Lesson 6.2.2 — Shapes of Scatterplots

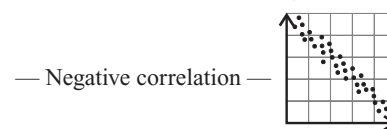
- As the x -value increases, the y -value increases



There is no obvious pattern connecting the x - and y -values of the data points

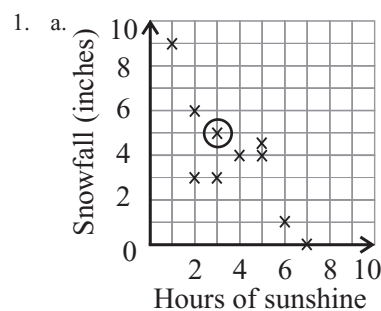


As the x -value increases, the y -value decreases



- a. **Positive correlation** — the cost of the pencils will increase with every pencil bought.
b. **No correlation** — these events are not related, so would not show any correlation.
c. **Negative correlation** — when traffic is heavy, drivers have to slow down to avoid accidents.

Lesson 6.2.3 — Using Scatterplots



- b. 1 in.
- a. 200 s
b. 1100 ft, 1200 ft, and 1400 ft

Section 7.1

Lesson 7.1.1 — Three-Dimensional Figures

- a. 5 faces
b. 9 edges
c. 6 vertices

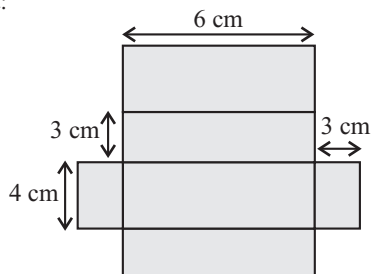
2. a. True
b. **False** — it has 5 side faces, and 2 bases, making 7 faces in total.
c. True

Lesson 7.1.2 — Nets

1. a. iii
b. iv
c. ii
d. i
2. A triangular prism.

Lesson 7.1.3 — Surface Areas of Cylinders and Prisms

1. Surface Area = $(2 \times \text{base area}) + \text{lateral area}$
2. First draw the net:



Now find the area of the net:

$$\begin{aligned}\text{Surface area} &= 2(6 \times 4) + 2(6 \times 3) + 2(4 \times 3) \\ &= 48 + 36 + 24 = \mathbf{108 \text{ cm}^2}\end{aligned}$$

Lesson 7.1.4 — Surface Areas & Perimeters of Complex Shapes

1. Split the shape into two rectangular prisms.
Prism A has an edge length of $(4 \times 10) + (2 \times 8) = 56 \text{ cm}$
Prism B has an edge length of $(4 \times 10) + (2 \times 8) = 56 \text{ cm}$
Three of prism A's 2 cm edges aren't edges of the combined shape. You need to subtract these from the edge length of both prisms.
So the total edge length of the shape is:
 $(56 - (3 \times 2)) + (56 - (3 \times 2)) = 50 + 50 = \mathbf{100 \text{ cm}}$
2. Split the shape into two rectangular prisms.
Prism A has an edge length of $(4 \times 6) + (4 \times 2.5) + (4 \times 2) = 42 \text{ in.}$
Prism B has an edge length of $(4 \times 2.5) + (8 \times 2) = 26 \text{ in.}$
Three of prism B's 2 in. edges aren't edges of the combined shape. You need to subtract these from the edge length of both prisms.
So the total edge length of the shape is:
 $(42 - (3 \times 2)) + (26 - (3 \times 2)) = 36 + 20 = \mathbf{56 \text{ in.}}$

Lesson 7.1.5 — Lines and Planes in Space

1. **Coplanar** — lines that intersect are always coplanar.
2. A, C, E

Section 7.2

Lesson 7.2.1 — Volumes

1. Thirteen small cubes have been joined together to make the shape. So its volume is **13 cubic units**.
2. Nine small cubes have been joined together to make the shape. So its volume is **9 cubic units**.

Lesson 7.2.2 — Graphing Volumes

1. a. About **43 cm³** (accept a near answer)
b. About **85 cm³** (accept a near answer)
c. About **2.15 cm³** (accept a near answer)
2. a. Volume of rectangular prism = length \times width \times height
 $= 4 \times 2 \times x = \mathbf{8x}$

b.

Prism height (x in.)	Prism volume (in ³)
0	0
1	8
2	16
3	24
4	32
5	40

Section 7.3

Lesson 7.3.1 — Similar Solids

1. Its side lengths will decrease.
2. a. **Yes** — in similar shapes, corresponding angles are always of equal measure.
b. **No** — the corresponding sides in shape B will be 2.5 times longer than in shape A.

Lesson 7.3.2 — Surface Areas & Volumes of Similar Figures

1. a. $4x \text{ cm}^2$
b. $8y \text{ cm}^3$
2. a. Surface area = $2(4 \times 2) + 2(3 \times 2) + 2(4 \times 3)$
 $= 16 + 12 + 24 = \mathbf{52 \text{ in}^2}$
b. If the prism is multiplied by a scale factor of 4, then its area will be multiplied by $4^2 = 16$.
 $52 \times 16 = \mathbf{832 \text{ in}^2}$

Lesson 7.3.3 — Changing Units

1. a. **100 cm** = 1 m
b. **12 inches** = 1 foot
c. **2.54 cm** = 1 inch
d. **0.3 m** \approx 1 foot

2. a. 1 in. = 2.54 cm
 $16 \text{ in.} = 16 \times 2.54 = 40.64 \text{ cm}$
 $14 \text{ in.} = 14 \times 2.54 = 35.56 \text{ cm}$
Rectangle area = $40.64 \times 35.56 = 1445.2 \text{ cm}^2$ (to 1 d. p.)
b. 1 foot $\approx 0.3 \text{ m}$
Surface area of cube $\approx 6(0.3 \times 0.3) = 0.5 \text{ m}^2$ (to 1 d. p.)
c. 1 in. = 2.54 cm
 $4 \text{ in.} = 4 \times 2.54 = 10.16 \text{ cm}$
Volume of cube = $(10.16)^3 = 1048.8 \text{ cm}^3$ (to 1 d. p.)
d. 1 foot $\approx 0.3 \text{ m}$
 $4 \text{ feet} \approx 4 \times 0.3 = 1.2 \text{ m}$
 $16 \text{ feet} \approx 16 \times 0.3 = 4.8 \text{ m}$
Volume of cylinder $\approx (\pi \times 1.2^2) \times 4.8$
 $= 1.44\pi \times 4.8$
 $= 6.912\pi = 21.7 \text{ m}^3$ (to 1 d. p.)

Section 8.1

Lesson 8.1.1 — Percents

1. a. $\frac{7}{100} = 7\%$
b. $\frac{18}{100} = 18\%$
c. $\frac{100}{100} = 100\%$
d. $\frac{86}{100} = 86\%$
2. a. $50\% = \frac{50}{100} = \frac{1}{2}$
b. $20\% = \frac{20}{100} = \frac{1}{5}$
c. $3\% = \frac{3}{100}$
d. $12\% = \frac{12}{100} = \frac{6}{50} = \frac{3}{25}$

Lesson 8.1.2 — Changing Fractions and Decimals to Percents

1. a. $0.5 \times 100 = 50\%$
b. $0.9 \times 100 = 90\%$
c. $0.25 \times 100 = 25\%$
d. $0.1 \times 100 = 10\%$
2. a. $0.87 \times 100 = 87\%$
b. $0.03 \times 100 = 3\%$
c. $0.17 \times 100 = 17\%$
d. $0.51 \times 100 = 51\%$
e. $1.45 \times 100 = 145\%$
f. $5.01 \times 100 = 501\%$
g. $7.68 \times 100 = 768\%$
h. $87.1 \times 100 = 8710\%$

Lesson 8.1.3 — Percent Increases and Decreases

1. a. $72\% \text{ of } 100 = \frac{72}{100} \times 100 = 72$
So the total = $100 + 72 = 172$
b. $30\% \text{ of } 10 = \frac{30}{100} \times 10 = 3$
So the total = $10 + 3 = 13$
c. $60\% \text{ of } 80 = \frac{60}{100} \times 80 = 48$
So the total = $80 + 48 = 128$
d. $10\% \text{ of } 6 = \frac{10}{100} \times 6 = 0.6$
So the total = $6 + 0.6 = 6.6$
e. $40\% \text{ of } 12 = \frac{40}{100} \times 12 = 4.8$
So the total = $12 + 4.8 = 16.8$
2. $15\% \text{ of } 22 = \frac{15}{100} \times 22 = 3.3$
So he should leave a total of = $\$22 + \$3.3 = \$25.30$

Section 8.2

Lesson 8.2.1 — Discounts and Markups

1. a. $\frac{50}{100} \times 48 = 24$,
so the sale price = $\$48 - \$24 = \$24$
b. $\frac{30}{100} \times 180 = 54$,
so the sale price = $\$180 - \$54 = \$126$
c. $\frac{35}{100} \times 62 = 21.7$,
so the sale price = $\$62 - \$21.70 = \$40.30$
d. $\frac{15}{100} \times 129 = 19.35$,
so the sale price = $\$129 - \$19.35 = \$109.65$
e. $\frac{23}{100} \times 56 = 12.88$,
so the sale price = $\$56 - \$12.88 = \$43.12$
2. a. $\frac{75}{100} \times 36 = 27$,
so the retail price = $\$36 + \$27 = \$63$
or $\frac{175}{100} \times 36 = \63

$$\begin{aligned} \text{b. } \frac{40}{100} \times 24 &= 9.6, \\ \text{so the retail price} &= \$24 + \$9.60 = \mathbf{\$33.60} \\ \text{or } \frac{140}{100} \times 24 &= \mathbf{\$33.60} \end{aligned}$$

Lesson 8.2.2 — Tips, Tax, and Commission

$$\begin{aligned} 1. \quad \text{a. } \frac{10}{100} \times 42.50 &= \mathbf{\$4.25} \\ \text{b. } \frac{20}{100} \times 42.50 &= \mathbf{\$8.50} \\ \text{c. } \frac{16}{100} \times 42.50 &= \mathbf{\$6.80} \\ 2. \quad \text{a. } \frac{115}{100} \times 27 &= \mathbf{\$31.05} \\ \text{b. } \frac{120}{100} \times 85 &= \mathbf{\$102} \end{aligned}$$

Lesson 8.2.3 — Profit

$$\begin{aligned} 1. \quad \$4.35 - \$1.50 &= \mathbf{\$2.85} \\ 2. \quad \text{Expenses} = \text{Revenue} - \text{Profit} &= \$174,000 - \$82,000 = \mathbf{\$92,000.} \end{aligned}$$

Lesson 8.2.4 — Simple Interest

$$\begin{aligned} 1. \quad I \text{ is the interest earned, } P \text{ is the principal — the amount that} \\ \text{is borrowed or deposited and has the interest earned on it.} \\ 2. \quad \text{a. } \frac{4}{100} \times 5000 &= \$200 \\ \text{so over 3 years the interest will be } 3 \times \$200 &= \mathbf{\$600} \\ \text{b. } \frac{5}{100} \times 7800 &= \$390 \\ \text{so over 4 years the interest will be } 4 \times \$390 &= \mathbf{\$1560} \\ \text{c. } \frac{8}{100} \times 12,000 &= \$960 \\ \text{so over 2 years the interest will be } 2 \times \$960 &= \mathbf{\$1920} \end{aligned}$$

Lesson 8.2.5 — Compound Interest

$$\begin{aligned} 1. \quad \text{a. Using the compound interest formula:} \\ P = 3800, r = 5\% = 0.05, t = 0.25, n = 1 \\ A = P(1 + rt)^n = 3800(1 + (0.05 \times 0.25))^1 \\ = 3800(1.0125)^1 = \mathbf{\$3847.50} \\ \text{b. Using the compound interest formula:} \\ P = 3800, r = 5\% = 0.05, t = 0.25, n = 2 \\ A = P(1 + rt)^n = 3800(1 + (0.05 \times 0.25))^2 \\ = 3800(1.0125)^2 = \mathbf{\$3895.59 \text{ (to the nearest cent)}} \end{aligned}$$

$$\begin{aligned} \text{c. Using the compound interest formula:} \\ P = 3800, r = 5\% = 0.05, t = 0.25, n = 4 \\ A = P(1 + rt)^n = 3800(1 + (0.05 \times 0.25))^4 \\ = 3800(1.0125)^4 = \mathbf{\$3993.59 \text{ (to the nearest cent)}} \end{aligned}$$

$$\begin{aligned} 2. \quad \text{a. } A &= P(1 + rt)^n \\ &= 6000(1 + (0.04 \times 1))^1 \\ &= \mathbf{\$6240} \\ \text{b. } A &= P(1 + rt)^n \\ &= 6000(1 + (0.04 \times 1))^3 \\ &= \mathbf{\$6749.18 \text{ (to the nearest cent)}} \end{aligned}$$

Section 8.3

Lesson 8.3.1 — Rounding

1. a. 7
b. 164
c. 100
d. 48
e. 1
f. 71
g. 6
h. 424
2. a. 0.1
b. 9.9
c. 80.5
d. 7.0
e. 2.9
f. 67.1
g. 376.2
h. 4.9

Lesson 8.3.2 — Rounding Reasonably

1. $12 \div 8 = 1.5$, so he will need to buy **2** bags of apples to have enough.
2. $33 \div 2 = 16.5$, so she will need **17** tins.

Lesson 8.3.3 — Exact and Approximate Answers

1. a. $\sqrt{18}$ in.
b. $\sqrt{18} = 4$ in. to the nearest inch
2. a. $\sqrt{3.50^2 + 6.02^2} = \sqrt{48.4904}$ cm
b. $\sqrt{3.50^2 + 6.02^2} = \sqrt{48.4904} = 7.0$ cm (to 1 decimal place)

Lesson 8.3.4 — Reasonableness and Estimation

1. a. **No, this isn't reasonable** — she can't give \$1.60 to her 6 children as this will be more than the \$8 she has.
b. **No, this isn't reasonable** — you can't have 0.75 of a person.
2. $8 \div 0.45 = 17.8$ (to 1 decimal place), so she can afford 17 oranges.